On symplectic integration of the guiding-center equations in general 3D toroidal fields using GORILLA

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In Ref. [1], a method for quasi-geometric integration of the guiding-center equations in general 3D toroidal fields was introduced. Realized in the GORILLA code [2], this method reduces the set of guiding-center equations to a linear ODE set with piece-wise constant coefficients by approximating the guiding-center Lagrangian with a continuous piece-wise linear function of the coordinates. The latter is achieved by representing the electro-magnetic field quantities with a 3D linear interpolation within tetrahedral cells which can be built on the basis of the spatial discretization of edge plasma codes, in particular, of the kinetic neutral code EIRENE. Thus, direct data exchange with these codes is facilitated. Since this method is not limited by field topology, its primary target is to model edge plasmas of toroidal devices with a general 3D geometry. Originally, the method has been realized for the approximate, lowest order time dynamics such that the shape of the orbits in the phase space was corresponding to a Hamiltonian system but the evolution in time could lead to minor artifacts in dwell time averages which are required for the computation of the spatial distribution of macroscopic parameters (density, plasma flows, pressure tensor). In the present work, the accurate time dynamics outlined in [1] has been realized in GORILLA resulting in orbits fulfilling the Hamiltonian properties in the whole extended phase space. The correctness of time dynamics is implicitly verified by computing the 1st Poincaré invariant in the extended phase space and by directly demonstrating the preservation of the symplectic form by the GORILLA flow map. The symplectic feature of the present integration method is also shown analytically in the case that the linear ODE set is integrated exactly. In GORILLA, a polynomial expansion of this linear set is used, however, computer accuracy of the resulting guiding-center orbits is achieved by employing a mild refinement of the integration time step within a tetrahedral cell, if necessary.

Furthermore, the application of the method to guiding-center orbit computation using the spatial discretization of the EIRENE code for the WEST tokamak geometry is presented as well.

References

[1] M. Eder *et al*, Physics of Plasmas 27, 122508 (2020), `doi.org/10.1063/5.0022117`

[2] M. Eder *et al*, zenodo (2021), `doi.org/10.5281/zenodo.4593661`