Free-plasma-boundary solver for 2D ideal MHD equilibria with flow

G. F.-Torija Daza [1] (gonferna@fis.uc3m.es), J.M. Reynolds-Barredo [1] (jreynold@fis.uc3m.es), R. Sanchez [1] (rsanchez@fis.uc3m.es). A. Loarte [2], G. Huijsmans [3] .
1) Departamento de Física, Universidad Carlos III de Madrid, Leganés, Spain.
2) ITER Organization, Route de Vinon-sur-Verdon, 13067 St Paul Lez Durance, France.
3) Department of Applied Physics, Eindhoven University of Technology, PO BOX 513, 5600 MB Eindhoven, Netherlands.
Institute for Magnetic Fusion Research, CEA, 13108 St Paul Lez Durance, France.

uc3m Universidad Carlos III de Madrid

1. Introduction: MHD eqilibria and free-boundary codes.

1.1. 2D MHD equilibrium for Tokamaks.

Magneto-hydro-dynamics equilibria (MHD) is routinely required in many contexts in magnetically confined fusion plasmas:

a) Design of the configuration.
b) Interpretation of multiple diagnostic systems during their operation.
c) Consequences of adding or removing elements (such as control coils in tokamaks).
d) When making changes in its configuration (varying currents in the external coil set, modifying the location and strength of particle and heat sources, etc.).
e) Other ones…

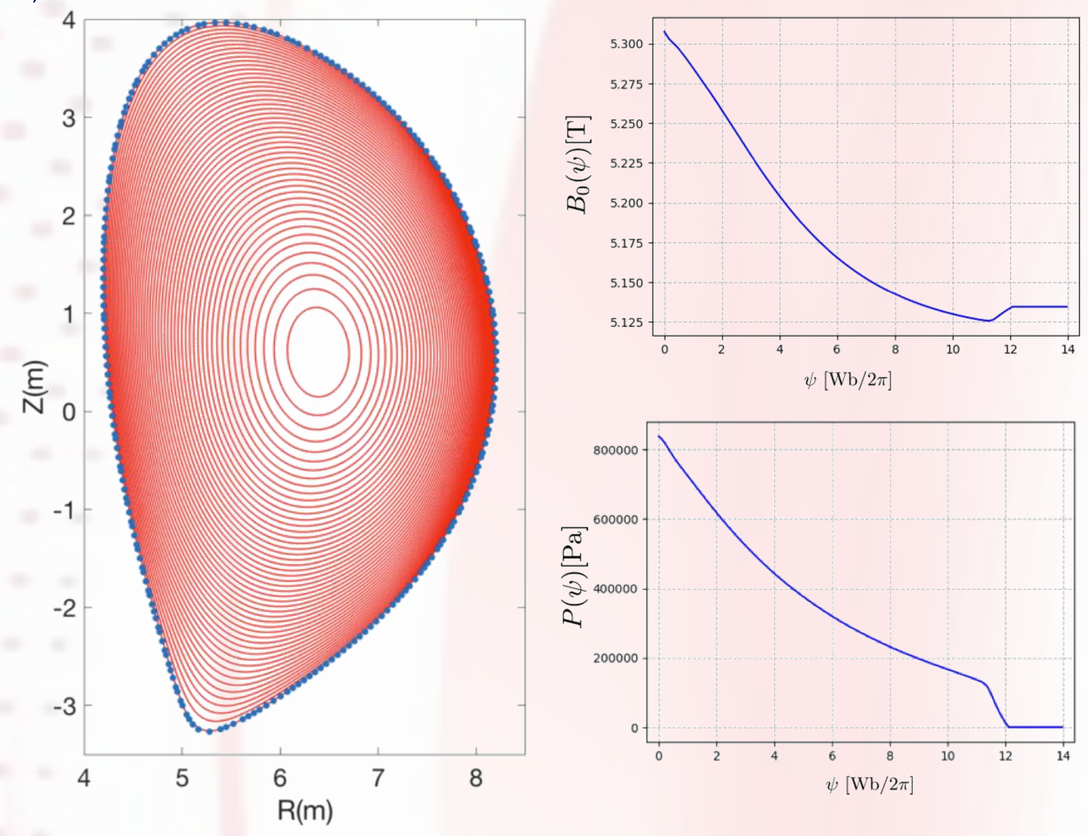


Figure: ITER equilibrium obtained from VMEC [1]. This is the baseline for all the displayed results.

1.2. Grad-Shafranov-Bernoulli system (GSB).

In the context of tokamaks
- Toroidal axisymmetry → good approximation in most experimental situations.
- Static case → Grad-Shafranov (GS) elliptic equation. In usual cyl. (R,φ,Z):

$$R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = -\frac{1}{2}\frac{dF^2}{d\psi} - \mu_0 R^2\frac{dp}{d\psi}$$

- Solved for the poloidal magnetic flux ψ.
- Pressure $p(\psi)$ and $F(\psi) = B_\varphi R$ are free functions.

In tokamak plasmas significant flows might be stablished due to different causes:
- External: biasing, neutral beam heating,…
- Internal: development of transport barriers,…

A) Flows must be introduced → Modified GS equation (Hameiri's version [2]):

$$\frac{1}{\mu_0}\nabla\cdot\left(\frac{(1-M_{Ap}^2)}{R^2}\nabla\psi\right) = -\frac{B_\phi}{\mu_0 R}\frac{dK}{d\psi} - (\mathbf{v}\cdot\mathbf{B})\frac{d\psi'_m}{d\psi} - \rho\frac{dH}{d\psi} - R\rho v_\phi\frac{d\Phi'}{d\psi} + \frac{\rho^\gamma}{\gamma-1}\frac{dS}{d\psi}$$

i) 5 free functions: $(K,\ \psi_m,\ H,\ \Phi,\ S)$
ii) Eq. is elliptic if $M_{A,p} < 1$.
iii) Now $\rho \not\equiv \rho(\psi)$ and $p \not\equiv p(\psi)$

B) Nonlinear nature → another equilibrium eq. is required, Bernoulli eq.:

$$H = \frac{1}{2}\left(\frac{\psi'_m}{\rho}\right)^2 B^2 - \frac{1}{2}(R\Phi')^2 + \frac{\gamma}{\gamma-1}S\rho^{\gamma-1}$$

1.3. Fix / Free boundary codes.

Fixed-Boundary (FXB): Computational boundary value ψ_b fixed from input to final result.
- Common element to actual the codes that manage to deal with all this complexity [3,4]

Free-Boundary (FRB): ψ_b is updated along the iterations and calculations.
- More realistic approximation. Difference is, usually, not negligible.

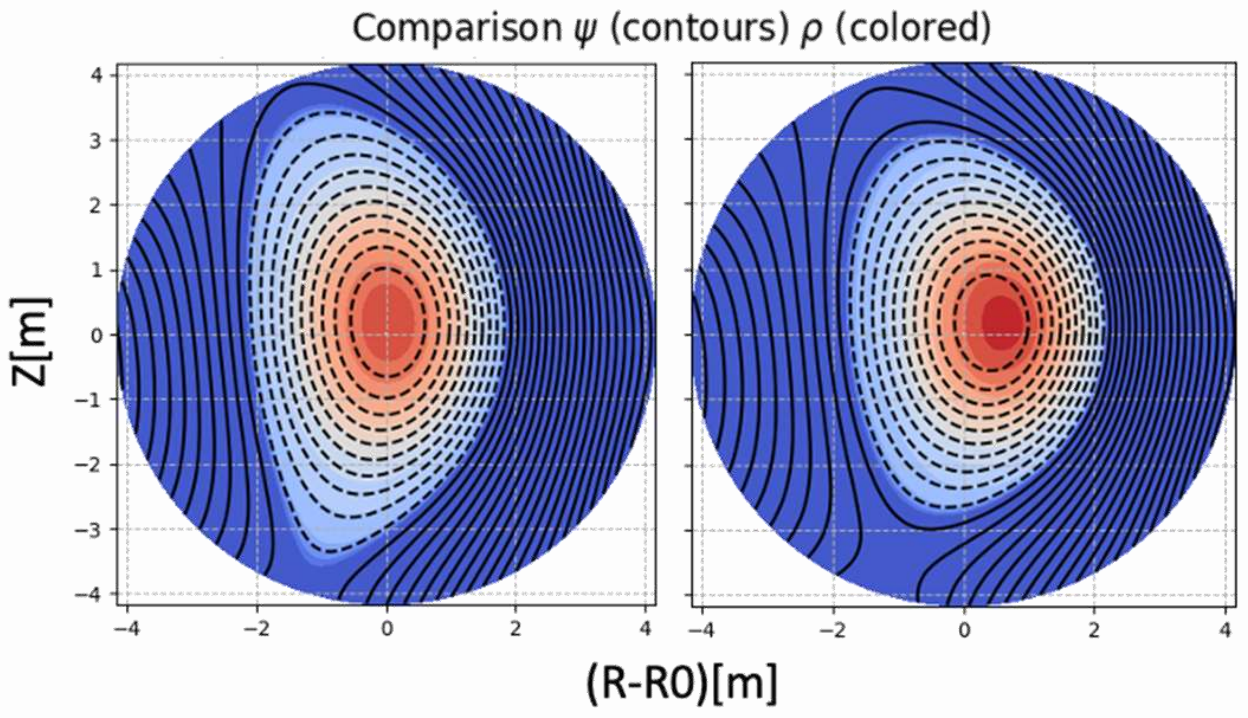


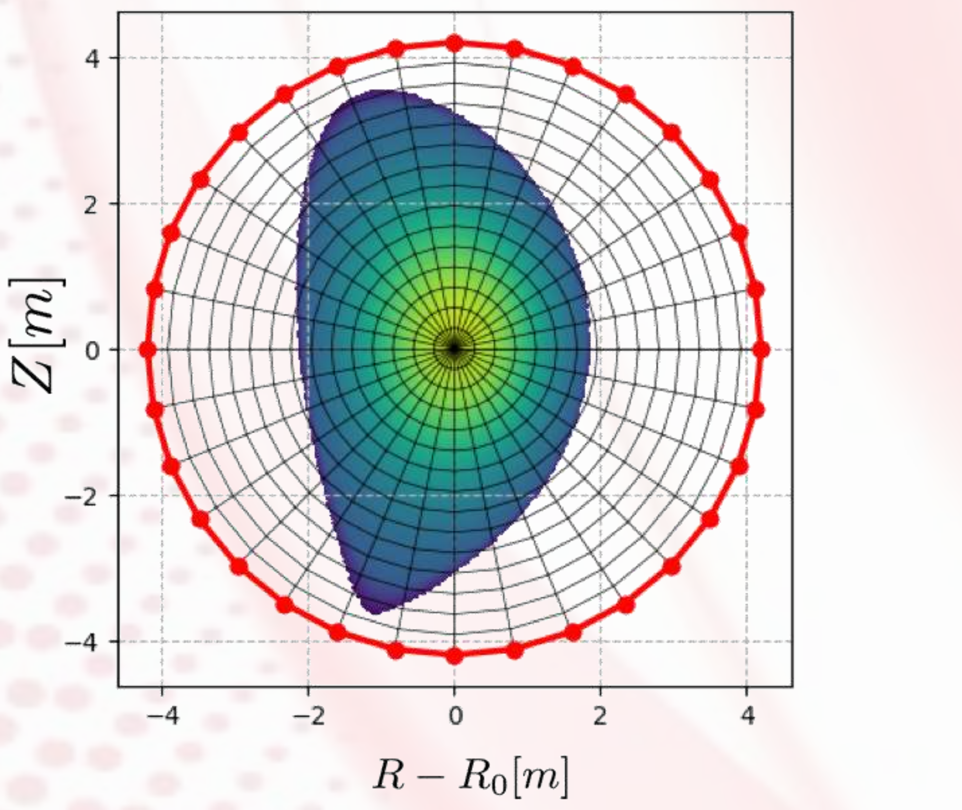
Figure: ITER free-boundary equilibrium: Static solution (left) vs toroidal flow profile (gf) with scaling factor $M_{tor} = 0.8$.

Fixed plasma boundary: In the context of FXB codes, the shape of the plasma is also prescribed a priori.
- Approximation shape of the plasma might be available → Reasonable approach.

Free plasma boundary: Discharge parameters variation → Plasma edge might change → Need of a code that let the edge to adapt to the force-balance eq.

2. The coupled free-boundary code.

-------- General Characteristics. --------
- Eulerian frame with coord. (r,θ,ζ) → Easier interaction with VMEC.
- Pseudo-spectral methods → exponential decreasing in Res (N_θ).
- 2nd order FD → quadratically decreasing in Res (M_r).
- R_0 may not coincide with the magn. axis.
- Comp. boundary in red $\psi_b = \psi(r = r_b,\ \theta)$.



-------- Iteration workout. --------

0) Initial solution $\psi^{(0)}(r,\theta)$
1) Modified GS (2) solver → $\psi^{(1)}(r < r_b,\ \theta)$
2) Bernoulli solver → $\rho^{(1)}$
3) Update computational boundary → $\psi_b^{(1)}$ → 1)

-------- Updating the booundary. --------

$$\psi_b = \psi_b^V + \psi_b^P$$

a) Vacuum field ψ_b^V: MAKEGRID code → Exterior coils contribution by Biot-Savart.
b) Plasma current contribution ψ_b^P: Solving $\nabla\times(\nabla\times A) = \mu_0 J^P$ for the vector potential A (most efficient way [5])

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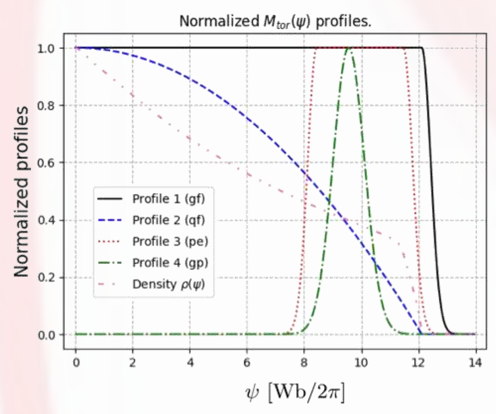
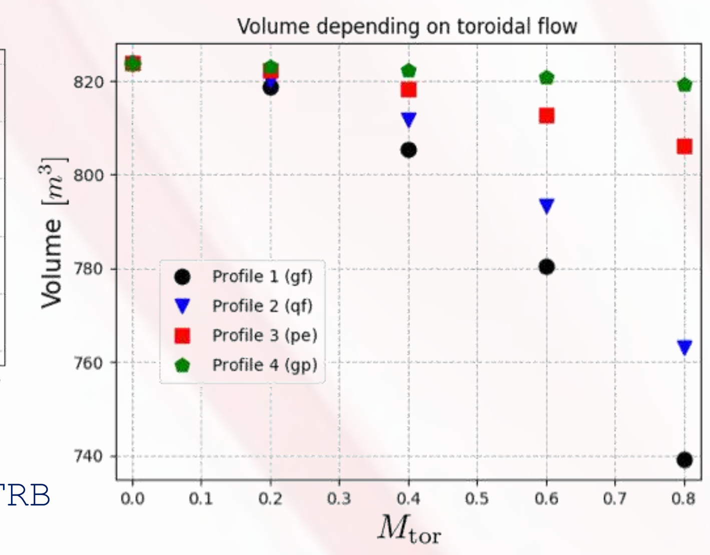
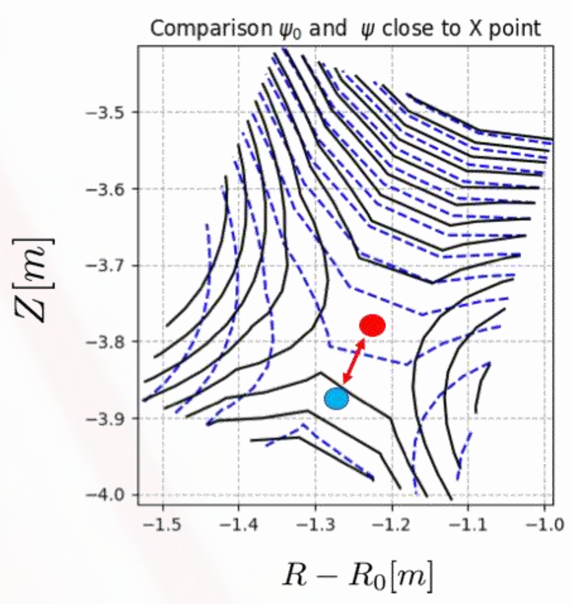
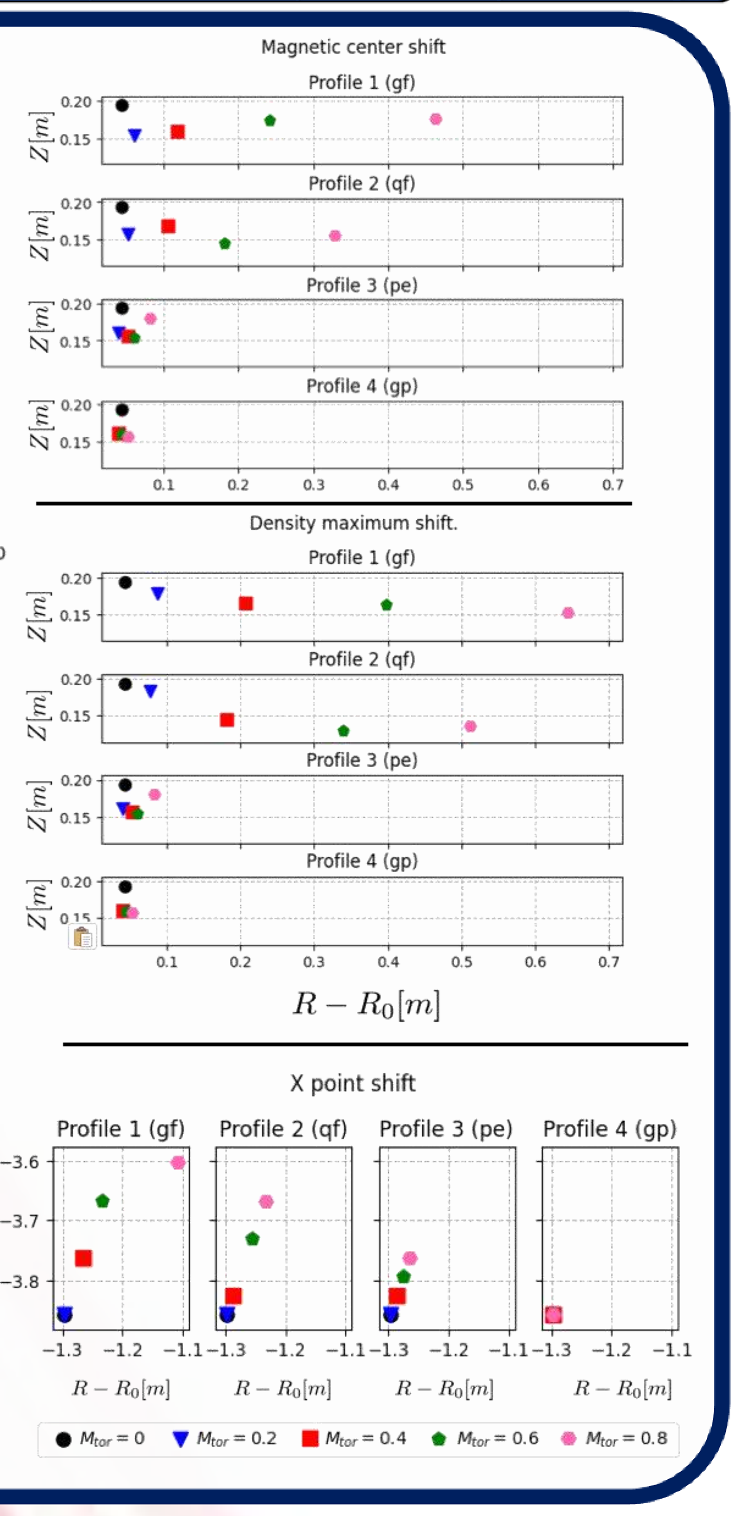
3. Showcase: ITER equilibrium example.









● **General set up.**
a) Initial solution: FRB static equilibrium solution from VMEC.
b) Profiles scaled by a factor $M_{tor} < 1$.
c) Purely toroidal flows.

● **Quantification possibilities.**
- Shape of the edge
- Confining volume Magnetic axis.
- Density maximum
- X-point
- Shapes (in general)
- Strong flows → deformation of plasma boundary → shifts of interior plasma points might be inaccurate in FXB.

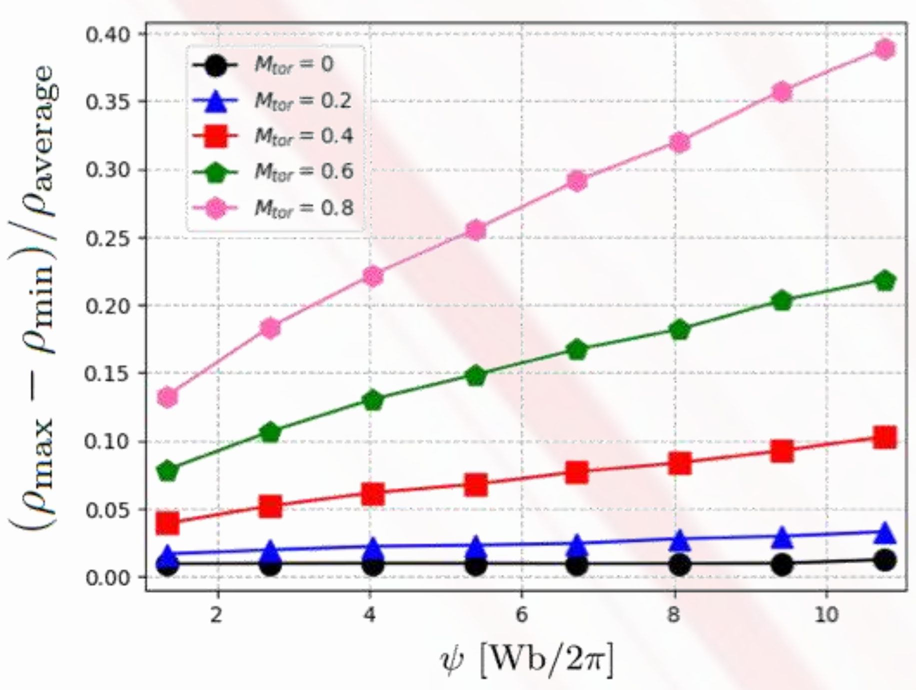


Figure: Variation of density along magnetic flux surfaces. Toroidal flow profile (gf) with scaling factor $M_{tor} = 0.8$.

4. Conclusions and future work.

Free-Boundary and Free-Plasma 2D code.
a) Successful coupling of
- FXB solver based in GSB system.
- Scheme, initially developed for static MHD equilibria in stellarators [6].

→ FRB 2D equilibria in the presence of plasma flows.

b) Tokamaks with arbitrarily complex coil sets can be easily treated within the framework by coupling the iterative solver to the VMEC and MAKEGRID codes.

Currently underway ---------------------------
a) Very elongated plasmas → generalized coordinates → arbitrary shape surfaces.
b) Study of devices with more influence of toroidal flows.
c) Presented scheme is not restricted to toroidal flows → similar study with poloidal flows.

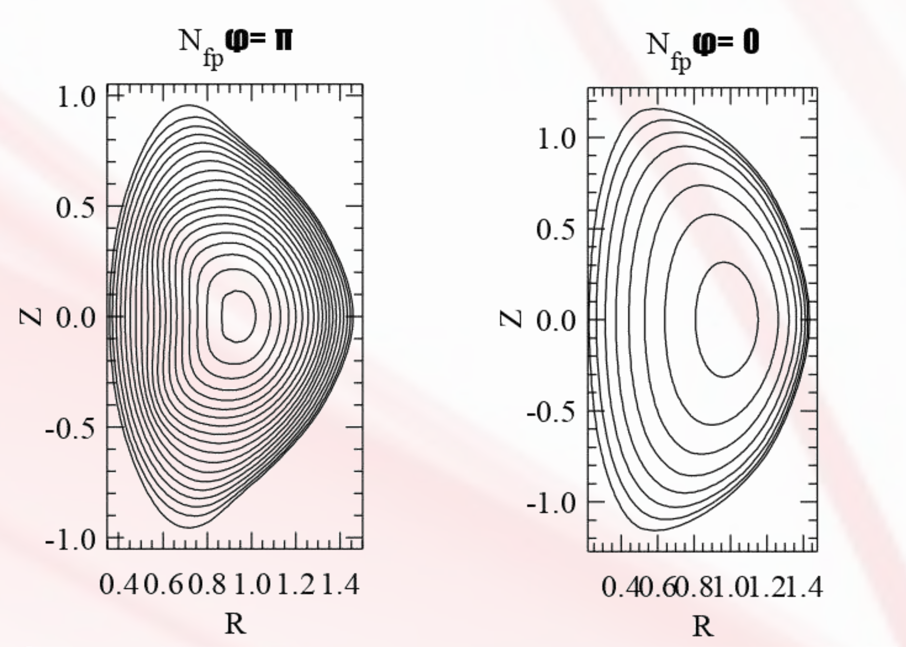


Figure: MAST (left) and NSTX (right) equilibria, where the coils are placed within the circle of the computational domain of our spectral code. Generalized coordinates are required in this case.

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