Theoretical and Numerical Study of Stimulated Raman Scattering in a Finite Plasma

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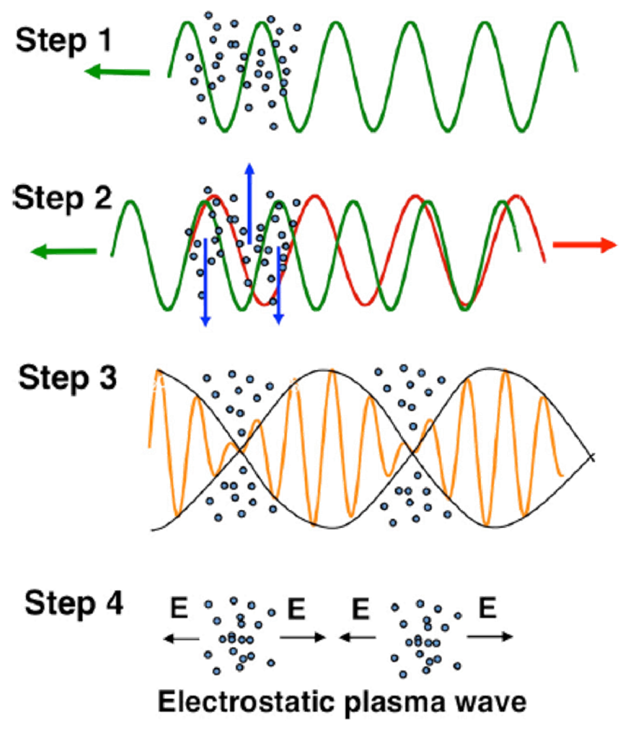
Introduction

The finite-length effect of Stimulated Raman Scattering(SRS) was investigated via both analytical theory and numerical simulation. An analytical dispersion relation to the absolute instability in a finite-length plasma system has been derived using two-scale-length expansion. The scaling of the linear growth rate of the scattered wave with the interaction length and the pump wave intensity has been examined and demonstrated. The three-wave simulation has been developed for studying the nonlinear behaviors of SRS, and the stability map obtained by the three-wave simulation reveals the non-oscillation zone, stationary oscillation zone, and non-stationary-oscillation zone of SRS. The depletion of pump power plays a crucial role in SRS, which results in the contraction of the field profile of the scattered wave. The start oscillation and the stationary oscillation can be achieved while the finite system reaches the energy balance condition. The theoretical results show good agreement with the simulation which is based on three-wave interactions. The Vlasov simulation code has also been developed for studying the kinetic effects in SRS, such as Landau damping and wave breaking in the finite-length system.

SRS Mechanism

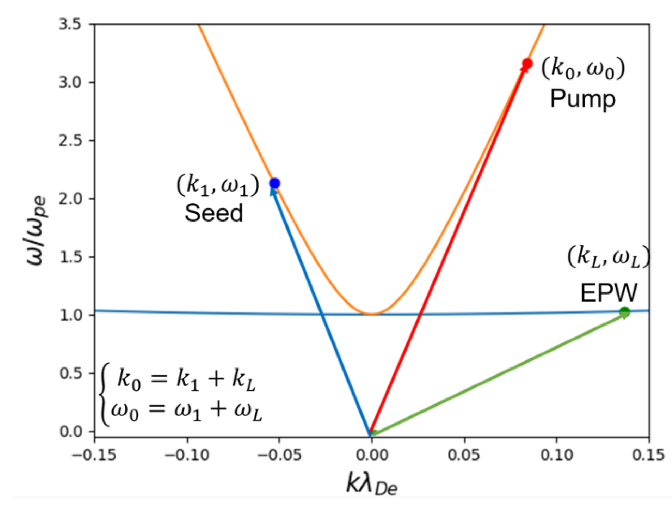


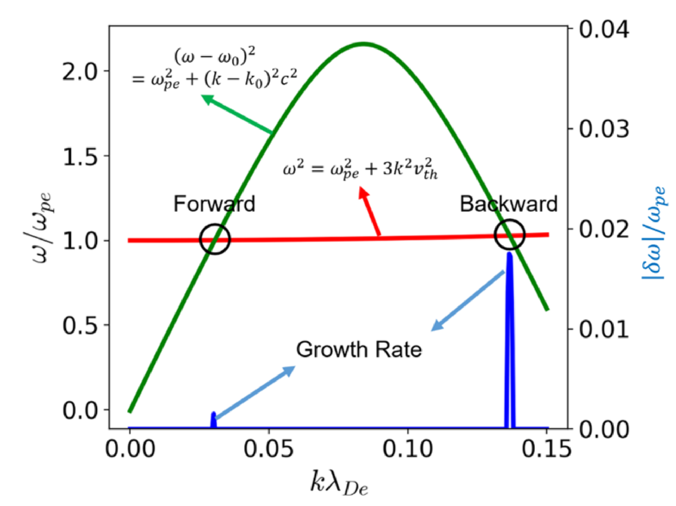
1. Pump Laser is injected into plasmas.
2. Electrons oscillate and the seed wave is induced
3. Pump wave and seed wave form a beat wave.
4. Beat wave enhances plasma waves, thus enhance the seed wave.

Form a positive feedback via parametric instability !

SRS Dispersion

$$\underbrace{(\omega^2-\omega_{pe}^2-3k^2v_{th}^2)}_{\text{Plasma Wave}}\underbrace{[(\omega-\omega_0)^2-(k-k_0)^2c^2-\omega_{pe}^2]}_{\text{Seed Wave}}=\underbrace{\frac{\omega_{pe}^2k^2v_{os}^2}{4}}_{\text{Coupling Strength}}$$





Coupling Strength (Related to Pump Intensity)

$v_{os}=\frac{qE_y}{m\omega_0}, E_y=\sqrt{2I_0/c\epsilon_0}$

Parametric Instability

$k_0=k_1+k_L$

$\omega_0=\omega_1+\omega_L$

Two-Scale-Length Expansion in Finite Length Plasma and System Set Up

1. Apply two-scale-length expansion on the dispersion relation

$$D(k,\omega)=(\omega^2-\omega_{pe}^2-3k^2v_{th}^2)[(\omega-\omega_0)^2-(k-k_0)^2c^2-\omega_{pe}^2]-\frac{\omega_{pe}^2k^2v_{os}^2}{4}=0$$

i.e. $D(k,\omega)\to\Delta(k+\delta k,\omega+\delta\omega)$

2. Apply absolute instability condition to find $\delta k=k_m$

$$\begin{cases}\dfrac{\partial\Delta}{\partial\delta k}=0\\ \Delta(k_m,\delta\omega)=0\end{cases}$$

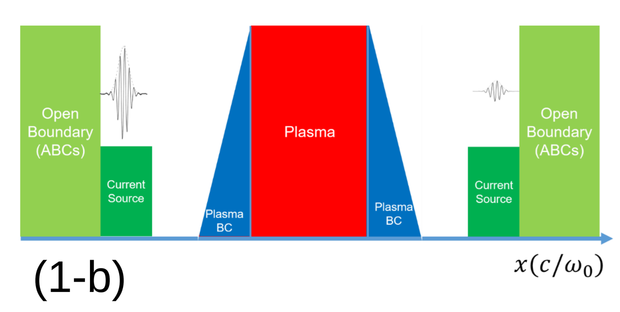
3. Let $\delta k=k_m-i\partial_z$ and apply the stationary field profile to find the growth rate

$$\Delta(k_m-i\partial_z,\delta\omega)A(z)=0\to\left[\Delta(k_m,\delta\omega)-\frac{1}{2}\partial_{k_m}^2\Delta\,\partial_z^2\right]A(z)=0$$

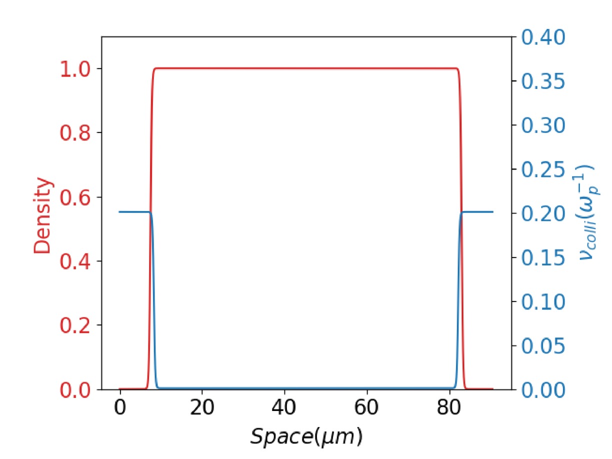
At there, we set plasma as a flatten region in the central part and density ramp at both side. And the density function can be shown as following equation.

$$n(x)=n_0(\tanh(m(x-d))+\tanh(-m(x-L+d)))$$

(1-a)



(1-b)



Plasma Parameter
Electron Temperature : $0.4keV$
Flatten Region : $10\sim100\mu m$
Plasma Density : $0.1n_c$
Laser Parameter
Wave Length : $800nm$
Laser Intensity : $1\times10^{13}\frac{W}{cm^2}\sim1\times10^{15}\frac{W}{cm^2}$

Vlasov 1D – $1\frac{1}{2}$V and Three-Wave-Interaction Simulation

Relativistic Vlasov Equation

$$\frac{\partial f_\alpha}{\partial t}=-\vec{v}\cdot\frac{\partial f_\alpha}{\partial\vec{x}}-\frac{q_\alpha}{m_\alpha}(\vec{E}+\vec{v}\times\vec{B})\cdot\frac{\partial f_\alpha}{\partial\vec{u}}+v_{\alpha i}(f_\alpha-f_{\alpha0}),\alpha\in\{\mathrm{i,e}\}$$

Fluid Equations

$$\frac{\partial P_{\alpha y}}{\partial t}=q_\alpha E_y$$
$$\frac{\partial P_{\alpha z}}{\partial t}=q_\alpha E_z$$
$$j_l=\Sigma_{\{l,\alpha\}}q_\alpha V_{\alpha l}$$

Maxwell Equations

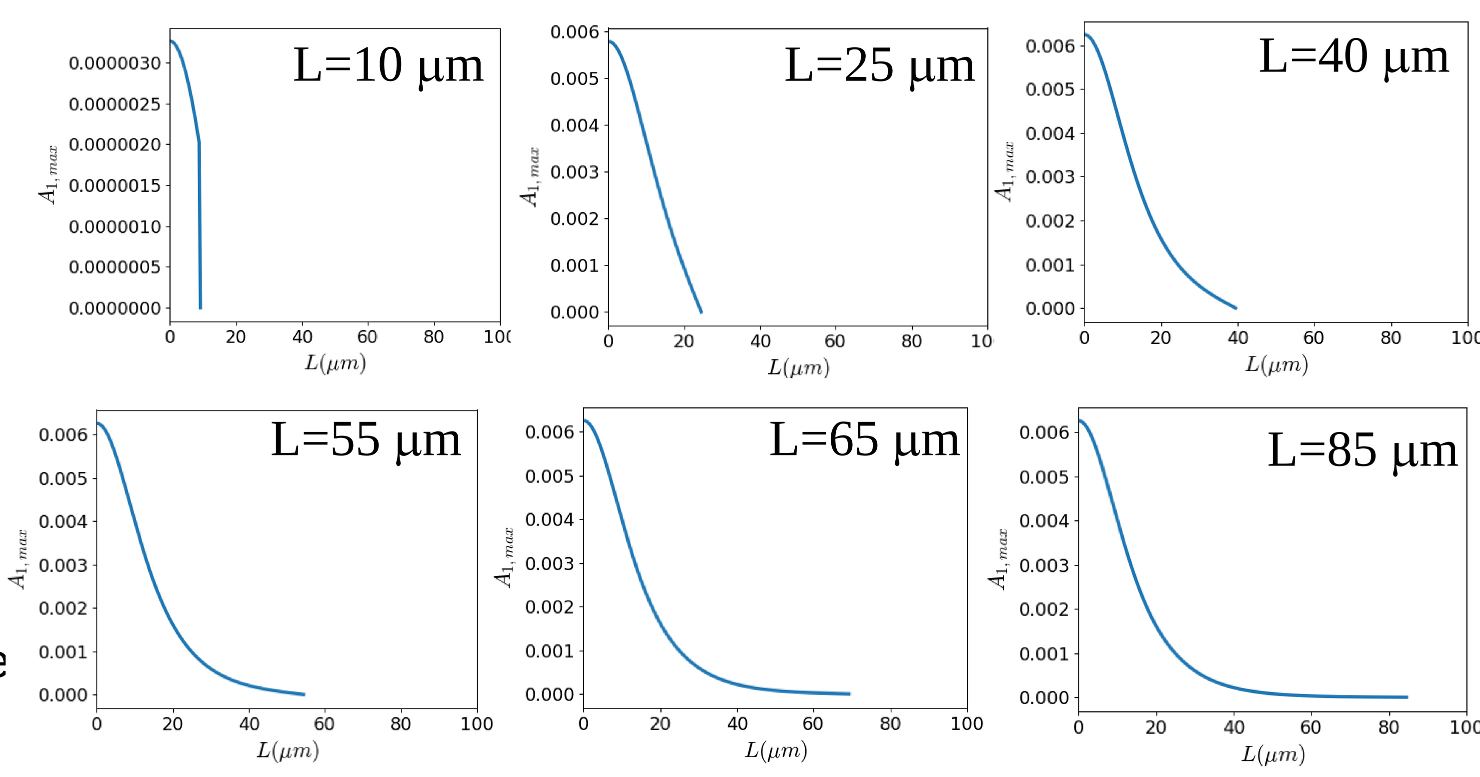
$$\nabla\cdot\vec{E}=\frac{\rho}{\epsilon_0}$$
$$\nabla\cdot\vec{B}=0$$
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\nabla\times\vec{B}=\mu_0(\vec{J}+\epsilon_0\frac{\partial\vec{E}}{\partial t})$$

$$\begin{cases}(\partial_{t'}+V'_{g0}\partial_{x'})\widetilde{A_0}=-\dfrac{1}{4}\dfrac{k'_L}{\omega'_0}\widetilde{E_L}\widetilde{A_1}\\ (\partial_{t'}-V'_{gs}\partial_{x'})\widetilde{A_1}=\dfrac{1}{4}\dfrac{k'_L}{\omega'_1}\widetilde{E_L^*}\widetilde{A_0}\\ (\partial_{t'}+V'_{gL}\partial_{x'})\widetilde{E_L}=\dfrac{1}{4}\dfrac{k'_L}{\omega'_L}\widetilde{A_0}\widetilde{A_1^*}\end{cases}$$

1^{st} order interpolation is used to evolve the field profile

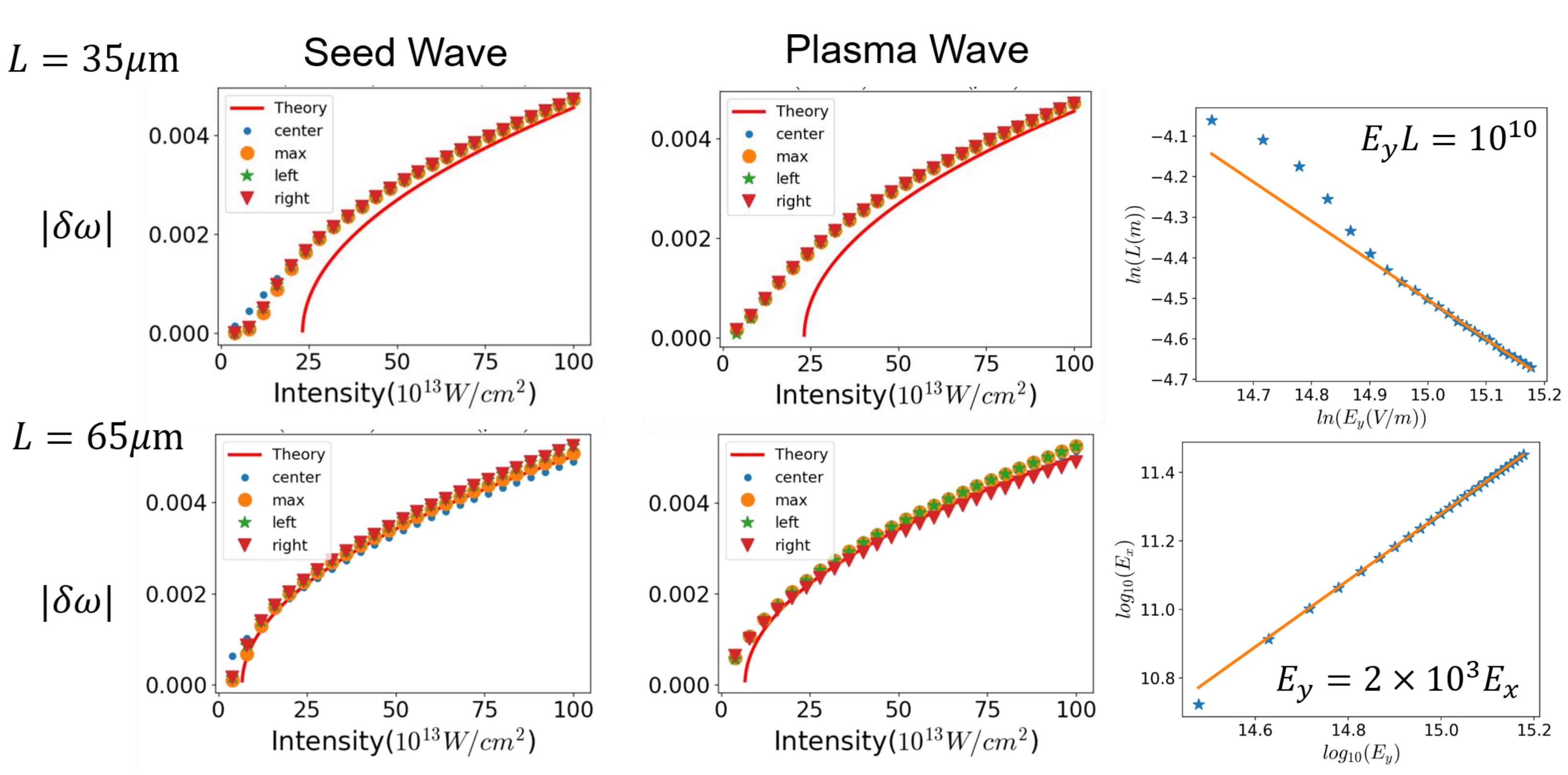
Real Space : WENO Method
Velocity Space : WENO Method
Time evolution : RK4+Predictor-Corrector method(Adam's Formula)

TWI Simulation Result



In the pump-depletion region, the scatter wave profile contracts to be within an effective length, which corresponds to the start oscillation length at a given pump intensity.

Linear Growth Rate and Start Oscillation Length



According to the simulation result, the scaling of the effective length with the pump wave intensity is

$$E_yL_{eff}=10^{10}$$

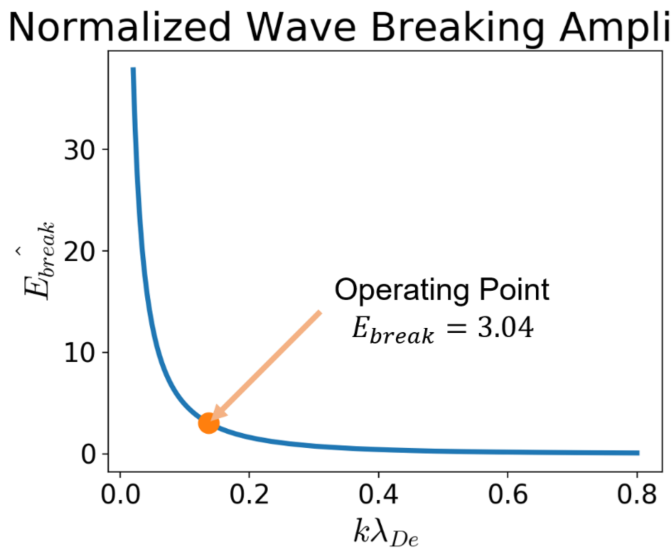
Or more convenience

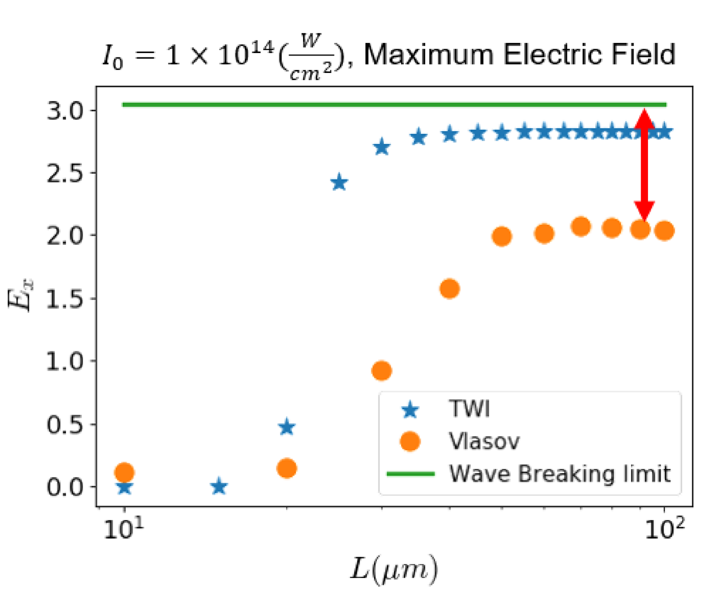
$$\sqrt{I_0\left(\frac{W}{cm^2}\right)}L_{eff}(\mu m)=4\times10^8$$

Another scaling law give us the relation between pump intensity and maximum longitudinal field amplitude

$$E_y=2\times10^3E_x$$

Under-Threshold Wave Breaking





In the Vlasov simulation, kinetic effect such as wave breaking, Landau damping is included. At there, the wave breaking amplitude is below the threshold predicted in infinite length theory. The finite length effect may induce the reduced threshold of wave breaking.

References

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