Onset of shearless transport barriers in a magnetically confined plasma

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An $\mathbf{E}\times\mathbf{B}$ drift wave transport model [1] was implemented to investigate the onset and break-up of shearless transport barriers (STBs) when considering a non-monotonic radial electric field profile for a magnetically confined plasma. These barriers were found by using the rotation number profile, since they are located in the profile's vanishing-derivative position, and analyzed by stroboscopic Poincaré phase portraits obtained by numerically integrating the motion equations. Considering as control parameters the amplitude of the electrostatic potential perturbation, non-resonant mode, and the radial position of the electric field extreme value, we found intervals of parameters values for which the barrier exists, breaks up or even bifurcates into two or three secondary shearless curves. Also, we found effective transport barriers related with stickiness regions which appear both before and after the STB breaks up. In general, we noted that the STB can emerge recurrently even if we are increasing the perturbation or displacing the electric field profile.

In first place, let us consider that the plasma particles are under the action of an electrostatic field, $\mathbf{E}(\mathbf{x},t)$, which can be decomposed into a radial mean part, $E_r(r)\hat{e}_r$, and a floating part, $\tilde{\mathbf{E}}(\mathbf{x},t) = -\nabla\tilde{\phi}(\theta,\varphi,t)$,

$$\mathbf{E}(\mathbf{x},t) = E_r(r)\hat{e}_r - \nabla\tilde{\phi}(\theta,\varphi,t). \tag{1}$$

The floating electrostatic potential, $\tilde{\phi}$, is regarded as a superposition of harmonic waves traveling in the poloidal and toroidal directions, θ and φ,

$$\tilde{\phi}(\theta,\varphi,t) = \sum_n \phi_n \cos(M\theta - L\varphi - n\omega_0 t - \alpha_n), \tag{2}$$

where M and L are their dominant wave numbers, respectively, ω_0 their fundamental angular frequency, ϕ_n the amplitude and α_n the phase for each perturbation mode.

Also, we assume that the plasma is magnetised by a magnetic field, $\mathbf{B}(\mathbf{x})$, which is assumed to be in the form of a screw pinch configuration, i.e, the $\mathbf{B}$ field has a θ and a φ components,

$$\mathbf{B}(\mathbf{x}) = B_\theta\hat{e}_\theta + B_\varphi\hat{e}_\varphi; \qquad B \approx B_\varphi \gg B_\theta, \tag{3}$$

for a tokamak approximated as a $2\pi R$ periodic cylinder, where R is the major radius of the torus. It means that $a/R = \varepsilon \ll 1$, with a the minor radius of the plasma.

Finally, for a test particle in the plasma, let us assume that its guiding center moves along the magnetic field lines with velocity $v_{\parallel}$ and an $\mathbf{E}\times\mathbf{B}/B^2$ drift,

$$\frac{d\mathbf{x}}{dt} = v_{\parallel}\frac{\mathbf{B}}{B} + \frac{\mathbf{E}\times\mathbf{B}}{B^2}. \tag{4}$$

On using two new variables, the action $I = (r/a)^2$ and the angle $\psi = M\theta - L\varphi$, the equation of motion (4) reduces to the 1.5-degrees-of-freedom dynamical system

$$\begin{aligned} \frac{dI}{dt} &= 2M\sum_n \phi_n \sin(\psi - n\omega_0 t - \alpha_n) \\ \frac{d\psi}{dt} &= \varepsilon v_{\parallel}(I)\frac{[M - Lq(I)]}{q(I)} - \frac{M}{\sqrt{I}}E_r(I), \end{aligned} \tag{5}$$

where $q(I) = rB/RB_\theta$ is the safety factor profile. In order to obtain (5), we adimensionalize the equation of motion (4) using the characteristic scales a, E_0 and B.

For the tokamak TCABR, we adopted the parameters $E_0 = 4.6$ kV/m, $B = 1.1$ T, $a = 0.18$ m and $\varepsilon \approx 0.3$. For the plasma profiles, we used the profiles proposed in [2, 3], namely, for the equilibrium radial electric field, $E_r(r) = 3\alpha(r/a)^2 + 2\beta(r/a) + \gamma$, with $(\alpha,\,\beta,\,\gamma) = (-0.563,\, 1.250,\, -1.304)$, for the safety factor, $q(r) = 1.0 + 3.0(r/a)^2$, and for the plasma toroidal velocity, $v_{\parallel} = -1.43 + 2.82\tanh(20.30r/a - 16.42)$.

Taking the perturbation angular frequency ω_0 as 62 rad $\cdot$ ms^{-1}, we noted that in the interval $I = [0.2, 1.4]$ only two modes are resonant, $n = 3$ and $n = 4$. So, we just considered three perturbation modes, as in [2, 3, 4], $\phi_3 = 1.0\times10^{-3}$, $\phi_4 = 0.12\times10^{-3}$, and a non-resonant mode, ϕ_2, which we varied from 0 to 8.5×10^{-3} with a stepsize 0.1×10^{-3}. Moreover, keeping α fixed, we considered the parameter $k = -\beta/(3\alpha)$, which is the radial position of the extreme value of E_r, and repeated the same procedure described below varying k with a stepsize 0.001.

Thus, we integrated numerically the set of equations (5) and constructed the $\psi \times I$ phase space drawing a point every period $T_j = j2\pi/\omega_0$, $j = 0, 1, 2, \ldots, N$. The existence of the STB was verified through the extreme values of the rotation number profile, Ω, which is determined by

$$\Omega = \lim_{N\to\infty}\frac{\psi_N - \psi_0}{N}. \tag{6}$$

From figures 1 and 2, we note that the STB prevents the chaotic transport outside the plasma, however, its appearance is recurrent. We see that there are several intervals for the control parameters, ϕ_2 and k, where the barrier can set on, break up or bifurcate, even if we are increasing the perturbation amplitude.

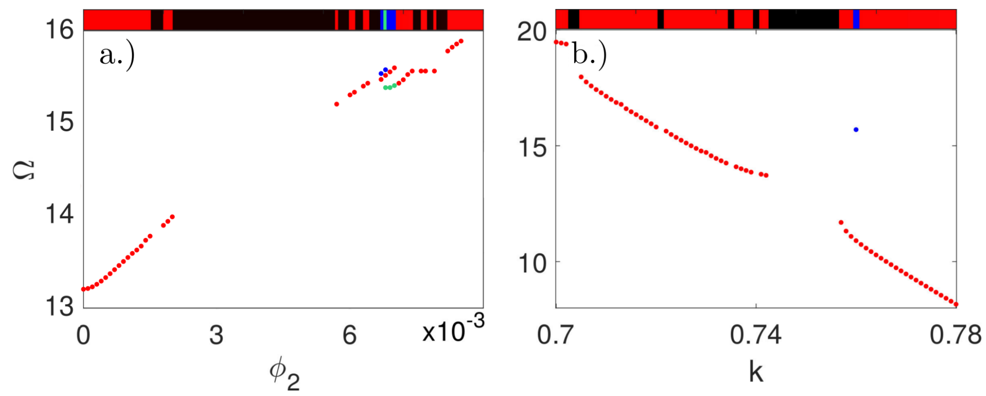

Figure 1: Rotation number of the STB as a function of the parameters (a) ϕ_2 and (b) k. Red (black) bars indicate the existence (non-existence) of shearless barriers. Blue and green bars indicate intervals with two or three barriers, respectively.

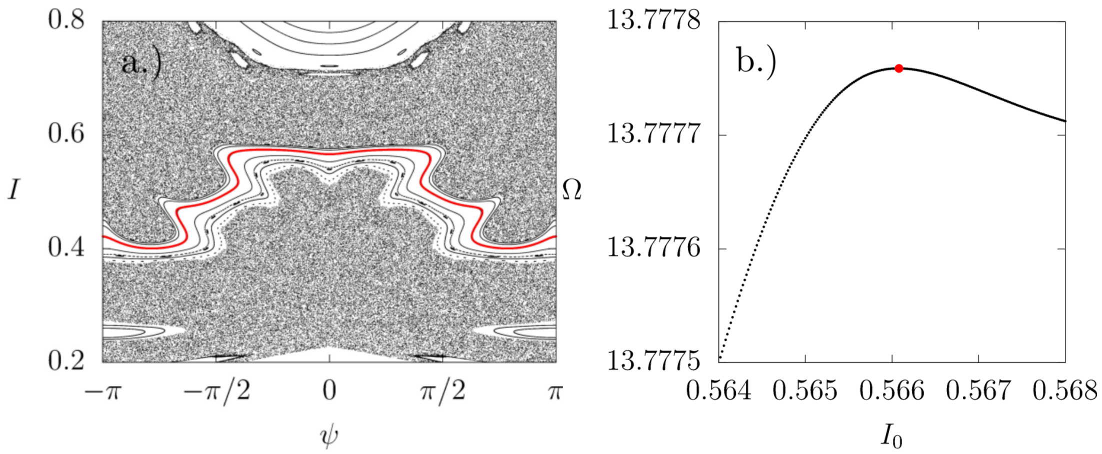

Figure 2: For $\phi_2 = 1.5 \times 10^{-3}$, (a) in the Poincaré section, we see a shearless transport barrier colored in red which disconnects two chaotic regions. (b) The barrier is determined through the maximum value of the rotation number profile.

In figure 3(a), a stickiness region appears next to the STB. It behaves as an effective barrier when the actual shearless curve is broken up, see figure 3(b). In this region, there are few crossings between the manifolds and, as a result, an orbit in the black (blue) region will spend a long time to cross to the blue (black) one.

Conclusions

We identified intervals where the barrier exists, is broken up or bifurcates into two or three additional barriers, for the parameters ϕ_2 and k.

We verified that shearless transport barriers offer a strong resistance to the chaotic transport even after they break up. In this process, a stickiness region can appear trapping the chaotic orbit for a long time.

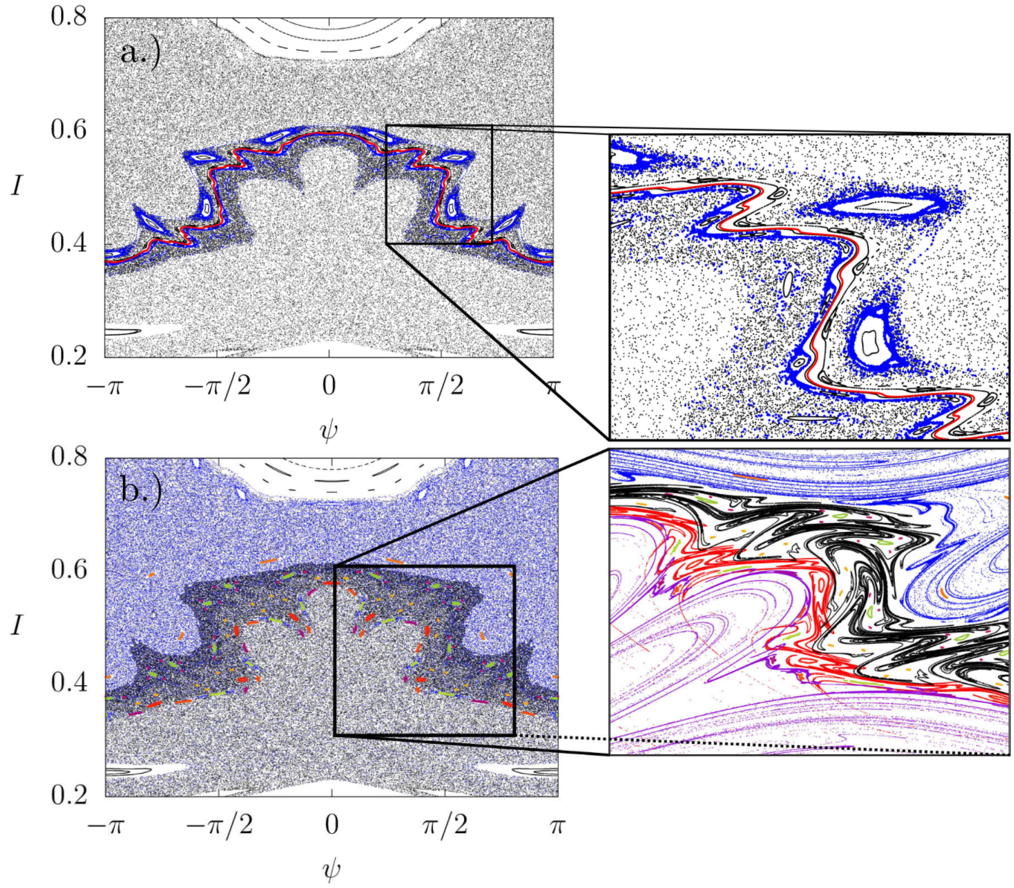

Figure 3: (a) For $\phi_2 = 2.0 \times 10^{-2}$, a stickiness region appears before the break-up of the barrier in (b) for $\phi_2 = 2.1 \times 10^{-3}$. In panel (b), we plot the stable (black and purple) and the unstable (red and blue) manifolds related with the stickiness.

We studied the dynamics of the stickiness region making use of the unstable and stable manifolds. We found that, in this region, crossings between manifolds barely occur, leaving few routes to the orbit to travel across the remnant of the barrier.

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