Nonlinear growth of Rayleigh Taylor Instability single mode structure under the effect of collision with a second fluid.

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CONTEXT AND OBJECTIVE

The ionosphere[1]

The ionosphere is a partially ionised gas that envelops earth and can be seen like the interface between the atmosphere and space.

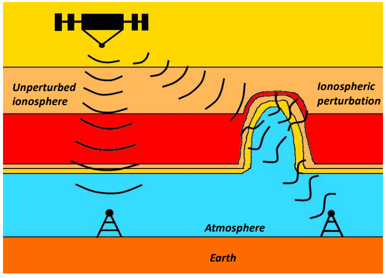

Figure 1: Schematic of a low-density plasma bubble rising in the ionosphere. This plasma bubble, by creating smaller irregularities, disturbs the signals received from satellites.

Characteristics :

- **Low degree of ionisation** $\alpha = \frac{n_i}{n_i+n_n} \approx 0{,}01$
- Low temperature : $T = 10^3$ K (compare to fusion plasma with $T = 10^8$ K)
- Peak of plasma density around 400 km with $n_e = 10^6$ cm^{-3}

➡ **Different irregularities can disturb high-frequency communications between the earth and satellites.**

➡ Radio waves are reflected, which is useful for AM radio and long-distance communication.

Generalized Rayleigh-Taylor Instability

The Generalized Rayleigh-Taylor instability (GRTI) occurs between two fluids at rest, subject to external forces pointing from heavy to light fluid. Thus, any perturbation of an interface between **a heavy fluid** (with mass density ρ_h) and **a light fluid** (with mass density ρ_l) will result in a rising light bubble and a falling heavy spike (see Fig.2).

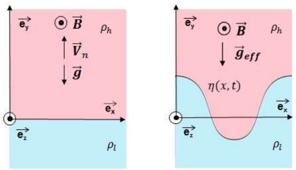

Figure 2: Scheme of the unperturbed (left) and perturbed (right) configuration.

The two destabilizing forces for ionosphere perturbations are:

- **Gravitational acceleration field:** $\boldsymbol{g} = -g\boldsymbol{e}_y$
- **Frictional drag force with neutral fluid:** $\boldsymbol{F}^n_{h(l)} = \rho_{h(l)}\nu_{in}\left(\boldsymbol{V}_n - \boldsymbol{V}_{h(l)}\right)$

where ν_{in} is the moment exchange collision frequency between ions and neutrals, $\boldsymbol{V}_n$ is the velocity of the neutrals and is assumed to be constant, $\boldsymbol{V}_n = U_0\boldsymbol{e}_y$. Note that we can remove the neutral velocities by including them in an effective gravity force, i.e. $g_{eff} = g - \nu_{in}U_0$.

The equation of the perturbed interface is given by $y = \eta(x,t)$.

Objective: Determine the impact of the frictional drag force with neutral fluid on the non linear growth of the GRTI.

ANALYTICAL NON-LINEAR MODEL

Hypothesis and method

Our non-linear study follow the work done by Goncharov[2] on the non-linear RTI.
We consider that the top of the bubble (resp. tip of the spike) is located at $x = 0$ and that the bubble (resp. spike) evolve with a **parabolic form**,

$$\eta(x,t) = \eta_0(t) + \eta_2(t)x^2,$$

where η_0 corresponds to the elevation along the y-axis of the top (resp. the position of the tip) of a bubble (resp. of a spike) and η_2 corresponds to the half value of the curvature of the top (resp. of the tip) of a bubble (resp. of a spike).

Moreover, we suppose that the fluids are **incompressible** ($\nabla \cdot \boldsymbol{V}_{h(l)} = 0$) and have an **irrotational** motion, so that the velocities derive from potentials $\phi_{h(l)}$ such as $\boldsymbol{V}_{h(l)} = -\nabla\phi_{h(l)}$. The velocity potentials for the heavier and lighter fluids obeying the Laplacian equation are assumed to be given by:

$$\phi_h(x,y,t) = a_1(t)\cos(kx)\, e^{-k(y-\eta_0(t))}, \quad y > 0,$$
$$\phi_l(x,y,t) = b_0(t)y + b_1(t)\cos(kx)\; e^{k(y-\eta_0(t))}, \qquad y < 0,$$

where k is the wave number of the perturbation, with $k = 2\pi/\lambda$. Injecting the parabolic bubble (resp. spike) shape and the velocity potentials into the kinetical boundary conditions and Bernoulli equations,

$$\frac{\partial\eta}{\partial t} - \frac{\partial\phi_h}{\partial x}\frac{\partial\eta}{\partial x} = -\frac{\partial\phi_h}{\partial y},$$
$$\left(\frac{\partial\phi_h}{\partial x} - \frac{\partial\phi_l}{\partial x}\right)\frac{\partial\eta}{\partial x} = \frac{\partial\phi_h}{\partial y} - \frac{\partial\phi_l}{\partial y},$$
$$\rho_h\left[-\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi_h)^2\right] - \rho_l\left[-\frac{\partial\phi_l}{\partial t} + \frac{1}{2}(\nabla\phi_l)^2\right] =$$
$$-g_{eff}(\rho_h - \rho_l)y + \nu_{in}(\rho_h\phi_h - \rho_l\phi_l) + f_h(t) - f_l(t),$$

and then, equating coefficient of order x^i ($i \leq 2$), we obtain a set of three ordinary differential equations describing our non-linear evolution of the top of the bubble.

Results

Dimensionless non-linear system

$$\frac{d\xi_1}{d\tau} = \xi_3$$
$$\frac{d\xi_2}{d\tau} = -\frac{1}{2}(6\xi_2 + 1)\xi_3$$
$$\frac{d\xi_3}{d\tau} = -\frac{6\xi_2 - 1}{D(\xi_2, r)}\left\{\frac{N(\xi_2,r)\xi_3^2}{(6\xi_2 - 1)^2} - 2(r-1)\xi_2 - C\xi_3\left[r(2\xi_2+1) - \frac{24\xi_2^2}{6\xi_2 - 1} + (2\,\xi_2 - 1)\frac{6\xi_2+1}{6\xi_2-1}\right]\right\}$$

With:

$$\xi_1 = k\eta_0,\quad \xi_2 = \eta_2/k,\quad \xi_3 = v_b\sqrt{\frac{k}{g_{eff}}},\quad \tau = t\sqrt{kg_{eff}},\quad r = \rho_h/\rho_l,\quad C = \nu_{in}/\sqrt{kg_{eff}}.$$

With $D(\xi_2, r) = 12(1-r)\xi_2^2 + 4(r-1)\xi_2 + (r-1)$ and $N(\xi_2, r) = 36(1-r)\xi_2^2 + 12(4+r)\xi_2 + (7-r)$.

In these equations, ξ_1, ξ_2, and ξ_3 are, respectively, the dimensionless (with rest to the wave number and effective acceleration field) **displacement, curvature, and velocity of the top of the bubble**, τ is the dimensionless time, r is the ratio of the mass densities, and C is a dimensionless parameter representing the collision drag over gravitational force. Following Goncharov's idea[2], the time evolution of **the spike** is obtained from the same set by making the transformations: $\xi_1 \to -\xi_1$, $\xi_2 \to -\xi_2$, $r \to 1/r$, and $g_{eff} \to -g_{eff}$.

Asymptotic Bubble Velocity

When $\tau \to +\infty$, the system converges toward an asymptotic solution where $d\xi_2/d\tau = 0$ and $d\xi_3/d\tau = 0$. This leads to a constant curvature, $\eta_2 = k/6$, and a constant velocity of the top of the bubble:

$$v_b = \frac{\nu_{in}}{k}\frac{1+2r}{6r}\left(\sqrt{1 + 12\frac{r(r-1)}{C^2(1+2r)^2}} - 1\right)$$

Classical regime[2] ($C \approx 0$)

$$v_b = \sqrt{\frac{\lambda g_{eff}}{6\pi}\frac{2A_t}{1+A_t}}$$

With $A_t = \frac{\rho_h - \rho_l}{\rho_h + \rho_l}$

Collisional regime[3,4] ($C \gg 1$)

$$v_b = \frac{g_{eff}}{\nu_{in}}\frac{2A_t}{3 + A_t}$$

COMPARISON WITH SIMULATIONS

Part of our work has been to simulate **the highly collisional configurations** ($C \gg 1$). We used **ERINNA[5]**, a two dimensionals (2D) eulerian code that solves the convection-diffusion and elliptic equations:

$$\frac{\partial\rho}{\partial t} - \frac{1}{B}\nabla\cdot(\rho\nabla_\perp\phi_E) - \kappa\Delta\rho = 0,$$
$$-\frac{1}{B}\nabla\cdot(\rho\nabla\phi_E) + \nabla\cdot(\rho\boldsymbol{V}_n \times \boldsymbol{e}_z) = 0,$$

where $\nabla_\perp = \left(-\partial_y, \partial_x\right)$, ϕ_E is the electric potential defined by $\boldsymbol{E} = -\nabla\phi_E$ with $\boldsymbol{E}$ the electric field following Ohm's Law $\boldsymbol{E} = -\boldsymbol{V}\times\boldsymbol{B}$ and κ is a diffusion coefficient.

The domain is defined by $x \in [0{,}12000]$ m and $y \in [0{,}12000]$ m The light fluid density is $\rho_l = 1\ \mathrm{kgm^{-3}}$ for $y > 6000$ m and ρ_h varies for $y < 6000$ m. A neutral wind is added as $\boldsymbol{V}_n = \boldsymbol{U}_0 e_y$ with $U_0 = 100\ \mathrm{ms^{-1}}$. The boundary condition is $\phi_E = 0$ at $x = 0$ or $x = 12000$ m and $\nabla\phi_E = 0$ at $y = 0$ and $y = 12000$ m. The perturbation is applied to the ion density as:

$$\rho(x,y) = \rho_s[1 \pm \beta\cos(k(x - x_0))]$$

where $\beta = 0{,}01$, $s \in \{h, l\}$, $x_0 = 6000$ m and the perturbation is negative for a bubble and positive for a spike.

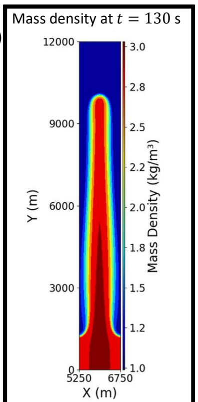

Figure 3: Simulation of plasma density with ERINNA centered around a spike, with $r = 3$.

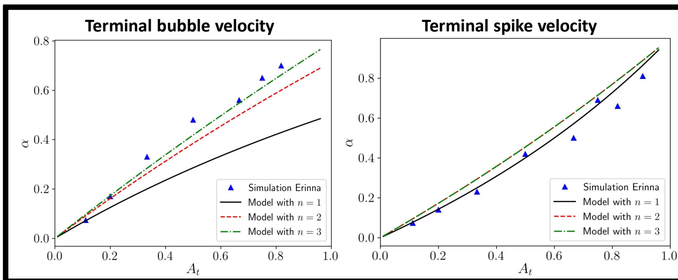

Figure 4: Normalized terminal velocity ($\alpha = v_b/(g_{eff}/\nu_{in})$) of the top of the bubble (left) and the tip of the spike (right) in the collisional regime as a function of the Atwood. Triangles represent simulation done with ERINNA. The straight lines represent our classical model, and the dashed and dot-dashed lines are our model extended with first and second harmonics, respectively.

Our model gives a **good approximation of the spike terminal velocity** in the collisional regime (see Fig.4).

For the **bubble terminal velocity**, the extension of our model by taking into account **higher harmonics** was necessary.
This is done by using the extended interface approximation and extended potentials solutions,

$$\eta(x,t) = \sum_{j=0}^{n}\eta_{2j}x^{2j}, \qquad \phi_h = \sum_{j=0}^{n} a_{2j+1}\cos[(2j+1)kx]\,e^{-(2j+1)k(y-\eta_0)}, \qquad \phi_l = \sum_{j=0}^{n} b_{2j+1}\cos[(2j+1)kx]\,e^{(2j+1)k(y-\eta_0)} + b_0 y.$$

CONCLUSION[6]

- ❖ Friction with a second ambient fluid was added to Goncharov's model, which gives a non-linear theory for the GRTI.
- ❖ Spike terminal velocity is well described by this model in the collisional range compared to the classical case.
- ❖ In the collisional regime, higher harmonics are necessary to obtain a precise bubble terminal velocity.

REFERENCES

[1] Michael C Kelley. *"The Earth's ionosphere: plasma physics and electrodynamics"*, (2009)
[2] V. N. Goncharov. *"Analytical model of nonlinear, single-mode classical Rayleigh-Taylor instability at arbitrary Atwood numbers"*, Physical review letters, (2002)
[3] E. Ott. *"Theory of Rayleigh-Taylor bubbles in the equatorial ionosphere"*, Journal of Geophysical: Space Physics, (1978)
[4] S. L. Ossakow and P. Chaturvedi. *"Morphological studies of rising equatorial spread F bubbles"*, Journal of Geophysical: Space Physics, (1978)
[5] F. Hermeline. *"A finite volume method for approximation of convection-diffusion equations on general meshes"*, International journal for numerical methods in engineering, (2012)
[6] Q. Cauvet, B. Bernecker, S. Bouquet, B. Canaud, F. Hermeline, and S. Pichon. *"Effect of collision with a second fluid on the temporal development of nonlinear, single-mode, Rayleigh Taylor instability"*, submitted, (2021)