Impact of trapped particles around infinitely-long electron-emitting objects in Maxwellian plasmas

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I) Introduction

The description of plasma sheaths is important in many interesting applications in plasma physics, including **emissive probes**, **charging** of bodies such as **spacecraft** or **dust grains**, and current exchange at the cathodic segment of **Low Work-function electrodynamic Tethers** (LWTs). The present work assesses the impact of the population of trapped particles on the structure of the sheath developing around a *two-dimensional electron-emitting* object. This is done by comparing the results of three numerical tools:

1. SVPS: a *Stationary Vlasov-Poisson Solver* based on the OMT for the sheath around infinitely long cylinders [1]
2. CPIC: a *PIC* code for the simulation of plasma-material interface with *curvilinear* geometries [2]
3. NSVPS: a novel *Non-Stationary Vlasov-Poisson Solver* currently under development

The SVPS cannot compute self-consistently the population of trapped particles and set it equal to zero. CPIC and the NSVPS, which are both non-stationary codes, capture the particle trapping during the transient phase.

II) 2D model for emissive probes

A cylindrical probe of radius r_p is immersed at rest in a **collisionless**, **unmagnetised**, **Maxwellian** plasma with density n_0 and temperature T_α, with $\alpha = e$ ($\alpha = i$) identifying plasma electrons (ions). **Half-Maxwellian** electrons ($\alpha = em$) with density n_{em0} are **emitted** at the probe contour Γ, which is biased at ϕ_p with respect to the plasma. After introducing an adequate normalisation (see details in Ref. [1]), the sheath obeys the **Vlasov-Poisson system**

$$\frac{\partial f_\alpha}{\partial \tau} + \boldsymbol{v}_\alpha \cdot \nabla_{\boldsymbol{r}} f_\alpha - \frac{e_\alpha}{2\mu_\alpha} \nabla\phi \cdot \nabla_{\boldsymbol{v}} f_\alpha = 0,$$

$$\Delta\phi = -\rho \equiv n_e + \beta n_{em} - e_i n_i, \quad n_\alpha(\tau, \boldsymbol{r}) = \int_{-\infty}^{+\infty} f_\alpha(\tau, \boldsymbol{r}, \boldsymbol{v}) d\boldsymbol{v}_\alpha$$

The system involves the **non-dimensional parameters** r_p, $\delta_\alpha \equiv T_\alpha/T_e$, $\mu_\alpha = m_\alpha/m_e$, $e_\alpha = q_\alpha/e$, $\beta = n_{em0}/n_0$ and ϕ_p, and the **boundary conditions**

$$\phi(\Gamma) = \phi_P, \quad \phi(r \to \infty) \to 0$$

$$f_{em}(\Gamma, \boldsymbol{v} \cdot \boldsymbol{u}_n > 0) = f_{HM} \equiv \frac{2}{\pi} \exp\{-(\boldsymbol{v} \cdot \boldsymbol{v})\}, \quad f_{em}(r \to \infty, \boldsymbol{v}) = 0$$

$$f_{e,i}(r \to \infty, \boldsymbol{v}) \to f_M \equiv \frac{1}{\pi} \exp\{-(\boldsymbol{v} \cdot \boldsymbol{v})\}, \quad f_{e,i}(\Gamma, \boldsymbol{v} \cdot \boldsymbol{u}_n > 0) = 0$$

being $\boldsymbol{u}_n$ the normal unit vector to Γ pointing outward.

III) Stationary solver: neglecting trapped particles

- Stationary ($\partial f_\alpha / \partial \tau = 0$) Eulerian solvers neglect the population of trapped particles arising during the transient (green curve, top half of the inset)
- Trapped particles would alter the sheath (red curve, bottom half of the inset)

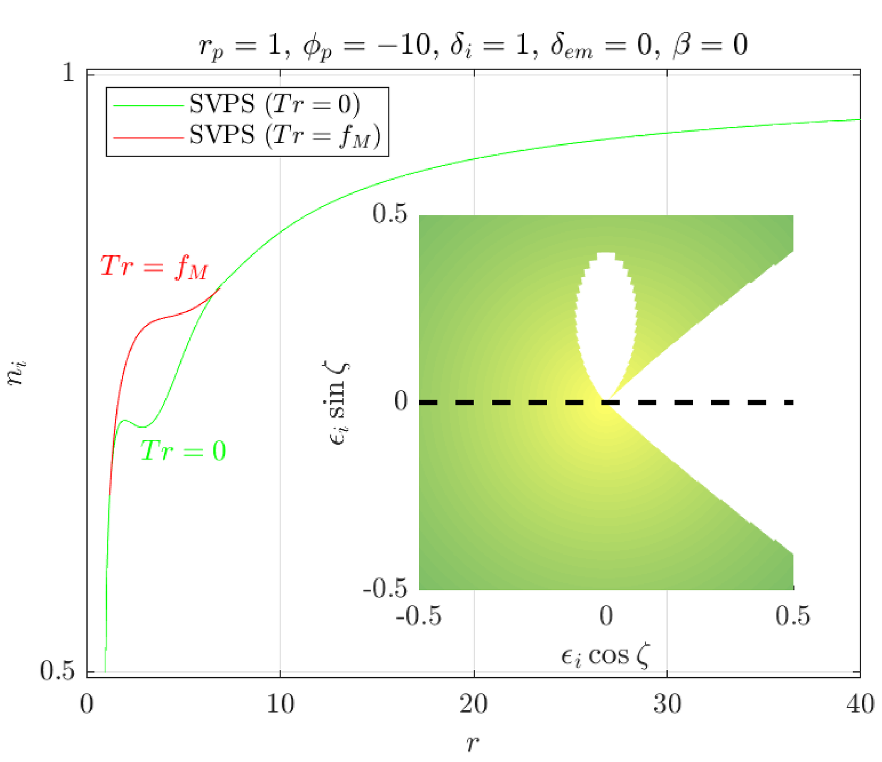

Figure 1: Density of attracted ions computed by setting the distribution of trapped particle to 0 (green) and f_M (red). Inset: detail of the distribution function in the $\epsilon_i \cdot \cos\zeta - \epsilon_i \cdot \sin\zeta$ plane, with e_i the total energy of the particles and ζ an angle in the velocity space.

IV) Trapped particles VS emission level

- For $r_p = 1$, $\delta_i = 1$, $\delta_{em} = 0.32$, $\mu_i = 1836$, $\phi_p = -10$ and the **initial condition** $f_\alpha(0, \boldsymbol{r}, \boldsymbol{v}) = f_0$, CPIC was used to compute self-consistently the plasma sheath.
- Starting from an empty domain (i.e., $f_0 = 0$), the resulting sheath resembles that predicted by the stationary solver (i.e., no trapping).
- If a Maxwellian plasma is already present inside the computational domain (i.e., $f_0 = f_M$), trapped particles starts appearing and the n_i profile for $\beta = 0$ shows a bump similar to that of Fig. 1.
- Their amount increases with the emission level, with n_i greater than greater than the unperturbed plasma density away from the probe, for $\beta = 30$.
- The potential well developing as a consequence of the strong emission at the probe boundary becomes less pronounced in presence of the trapped population. The current of emitted electrons is $\approx 20\%$ bigger than without trapping.

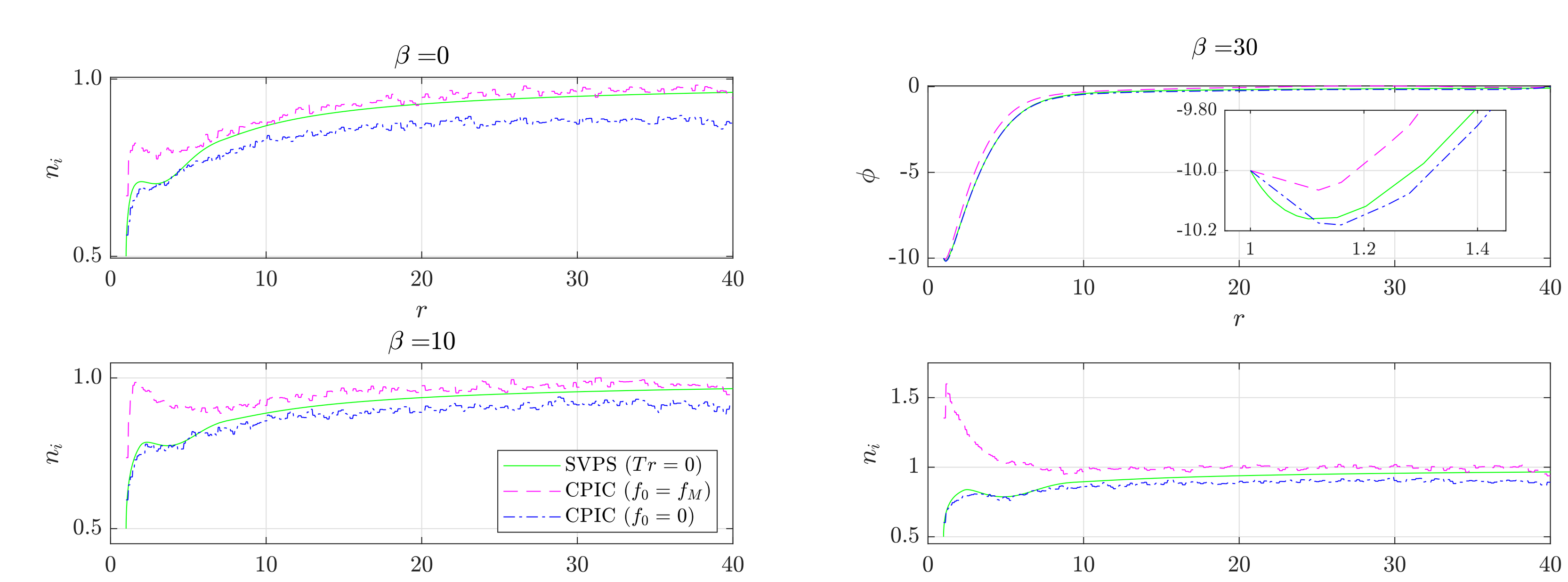

Figure 2: Density of attracted species for $\beta = 0$ (top left), $\beta = 10$ (bottom left), $\beta = 30$ (bottom right). Electrostatic potential profile for $\beta = 30$ (top right). Results shown for SVPS (solid green) and CPIC for $f_0 = 0$ (dash-dotted blue) and $f_0 = f_M$ (dashed magenta).

V) Backward semi-Lagrangian algorithm

In a collisionless plasma,

$$f_\alpha^{n+1}(\boldsymbol{r}, \boldsymbol{v}) = f_\alpha(\tau_n + \Delta\tau, \boldsymbol{r}, \boldsymbol{v}) = f_\alpha(\tau_n, \boldsymbol{r}^*, \boldsymbol{v}^*) = f_\alpha^n(\boldsymbol{r}^*, \boldsymbol{v}^*).$$

With $\Delta\tau > 0$, $(\boldsymbol{r}^*, \boldsymbol{v}^*) = (\boldsymbol{r} - \boldsymbol{v} \cdot \Delta\tau, \boldsymbol{v} + (e_\alpha/2\mu_\alpha)\nabla\phi \cdot \Delta\tau)$ is the origin of the characteristic passing through $(\tau_n + \Delta\tau, \boldsymbol{r}, \boldsymbol{v})$. The value of $f_\alpha^n(\boldsymbol{r}^*, \boldsymbol{v}^*)$ can be computed through interpolation from $f_\alpha^n(\boldsymbol{r}, \boldsymbol{v})$. A novel *backward* semi-Lagrangian code based on the approach here outlined is currently under development. Its features include

- Integration of the Vlasov characteristics with an hybrid, energy-conserving Crank-Nicolson method
- Multi-linear conservative Cloud-In-Cell (CIC) interpolation module

Preliminary results show that

- Particles with $e_i < 0$ (at steady-state) gets trapped during the transient
- Since a low order (linear) scheme is used for interpolation, a high resolution is needed to capture the solution correctly.

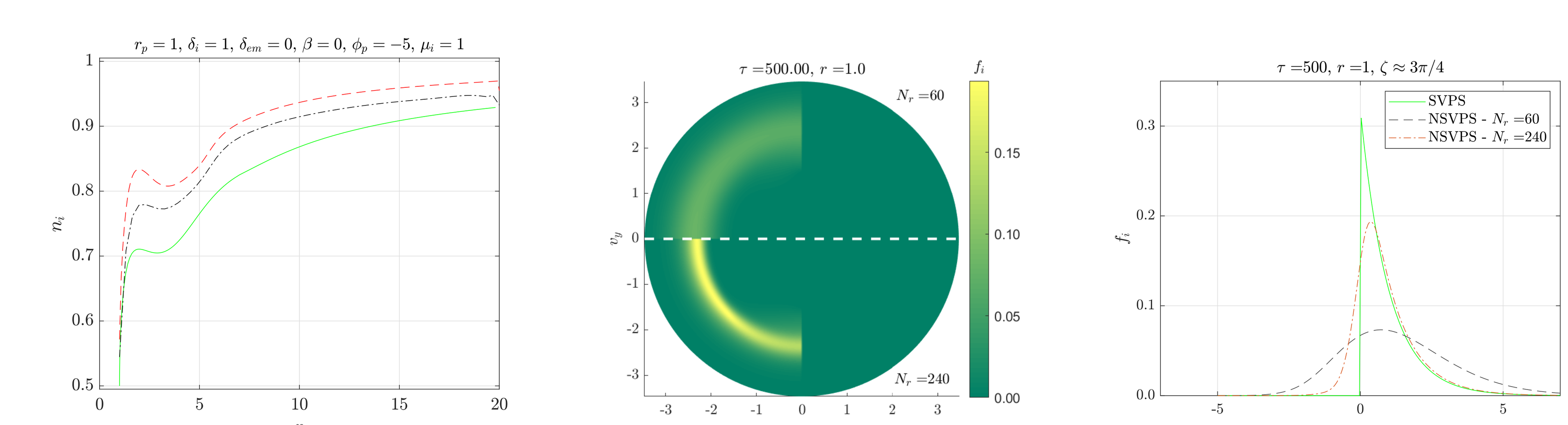

Figure 3: For $f_0 = f_M$ (NSVPS only): density of attracted species (left), map of $f_i(\Gamma)$ in the velocity space (middle), profile of $f_i(\Gamma)$ for a certain velocity angle (right). NSVPS results are shown for different N_r (i.e., number of nodes along r).

VI) Conclusions & Future activities

Main conclusions

- Comparing CPIC against SVPS showed that the final state depends on the initial conditions. The amount of trapped particles depends on the history of the system, as was also observed in the expansion of a collision-less plasma in a magnetic nozzle [3].
- Electron-emission enhances the population of trapped particles, which opposes the onset of the SCL regime
- Unlike CPIC, the NSVPS provides solutions free of statistical noise, but preliminary results suggest that a high order interpolation scheme or a fine grid is needed to capture correctly the strong gradients of f_α.

Next steps

- Extension of the analysis to i) a wider range of emission levels, ii) elliptic probes, iii) charging problems
- Implementation and verification of a high order interpolation method for the semi-Lagrangian solver

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References

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