Force-free Collisionless Current Sheets: A Systematic Method for Adding Asymmetries

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Current sheets are regions of space in which the current density is very localised and varies strongly in one spatial direction. Equilibrium particle distribution functions are known for force-free current sheets and lead to spatial density and temperature structures which are either constant in space or vary symmetrically. Recent observations of current sheets in the solar wind have shown systematic asymmetries in particle density and temperature while the pressure remains constant (further references in [1]). In this contribution we describe a systematic approach to finding distribution functions for this specific case. This mathematical foundation has been used to show why examples mentioned in Neukirch et al. (2020) [1] succeed and how it can be used to find new ones. The latter is not a straightforward process: even if a function satisfies all mathematical requirements, it can still be unreasonable in a physical sense.

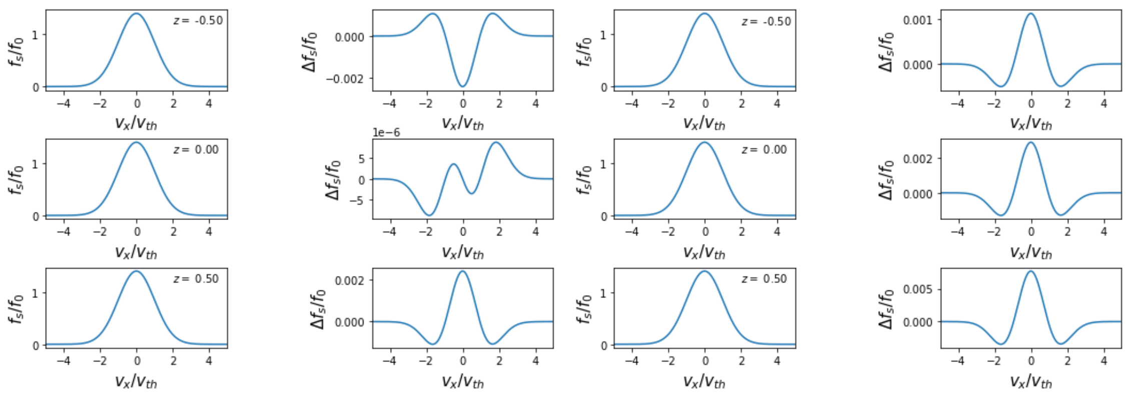

Figure 1: *Dependence of f_s and Δf_s on v_x (for $v_y = v_z = 0$) at three different positions $z/L = -0.5$ (top row), $z/L = 0.0$ (middle row) and $z/L = 0.5$ (bottom row). Full particle distribution function using g_1 with k_2 in first column and g_1 with k_3 in third column. For Δf_s alone using g_1 with k_2 in second column and g_1 with k_3 in fourth column. Here $\varepsilon = 0.01$ and $u_0/v_{th} = -3.9 \cdot 10^{-3}$.*

Due to the one-dimensional nature of the current sheet models one can find equilibrium distribution functions of the form

$$f \equiv f(H, p_x, p_y)$$

where $H = \frac{1}{2}mv^2 + q\phi$ is the Hamiltonian with $v^2 = v_x^2 + v_y^2 + v_z^2$ and $p_x = mv_x + qA_x$ and $p_y = mv_y + qA_y$ are the canonical momenta in x and y direction. We start from a known distribution function for the force-free Harris sheet [2]. In order to model the observed asymmetric variations in density and temperature Neukirch et al. (2020) [1] have added a term Δf to the force-free Harris sheet distribution function as seen in Harrison and Neukirch (2009) [3] and Neukirch et al. (2009) [4] leading to

$$f = f_{ff} + \Delta f$$

where Δf adds a spatial asymmetry to the number density. However, Δf does not contribute to the current density, i.e. has to satisfy

$$\int \Delta f d^3v \neq 0,$$

and

$$\int \mathbf{v}\Delta f d^3v = 0. \tag{1}$$

As seen in Neukirch et. al (2020) [1] one possible class of functions for Δf is of separable form, such that $\Delta f \equiv \Delta f(H, p_x) = g(H)k(p_x)$ with $\partial \Delta f / \partial p_x = g(H)k'(p_x)$. Different approaches to find suitable pairs of $g(H)$ and $k(p_x)$ exist of which we have focussed on an ansatz using the Fourier transformation. Let $\phi = 0$ and $G \in C^2(\mathbb{R})$ with $g(H) = G''(H)$. Let F be an appropriate function with $F(qA_x - p_x) = G\left((p_x - qA_x)^2/(2m)\right)$, then condition Eq. (1) can be written as

$$0 = \int_{-\infty}^{\infty} k'(p_x)F(qA_x - p_x)dp_x$$

which is an integral equation of convolution type. Therefore, its Fourier transform is given by the product of the Fourier transforms of k' and F individually:

$$\hat{k'}(u) \cdot \hat{F}(u) = 0 \tag{2}$$

where the Fourier transform is indicated by ^. Neukirch et al. (2020) [1] have proposed combinations of $k_1(p_x) = C_1 p_x$, $k_2(p_x) = \frac{1}{\omega}\sin(\omega p_x)$ and $k_3(p_x) = \frac{1}{\omega}\exp(\omega p_x)$ with $g_1(H) = K(e^{-aH} - ce^{-bH})$ and $g_2(H) = K(a - bH)e^{-bH}$. The Fourier transform approach can be used to show that these functions satisfy the conditions specified above. Figure 1 shows the dependence on v_x of the full particle distribution functions and its additional contribution for specific values of z. Figure 2 shows the corresponding asymmetric density and temperature profiles, which do not seem to be consistent with observations (see [1]).

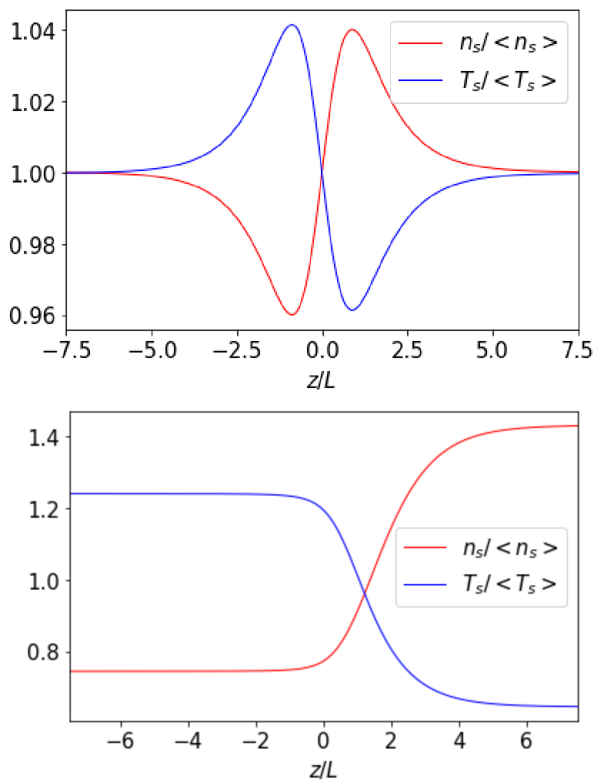

Figure 2: *Asymmetric density and temperature profiles resulting from the theoretical model for k_2 on the top and k_3 on the bottom (identical for with g_1 and g_2 in both cases). ($\varepsilon = 0.01$)*

One can use condition Eq. (2) to attempt to find new pairs of functions. Wilson and Neukirch (2011) [5] have used a delta function as $g(H)$, so we chose $\hat{k}'$ such that Eq/ (2) vanishes. This did not lead to any analytically available functions for k and major problems of this example are caused by the derivatives of the delta function that can lead to the distribution function not being zero. We integrate $g = G''$ twice and use this to rewrite the integral condition on k and g as

$$\int_{-\infty}^{\infty} k'(p_x)(H_0 - \overline{H}_{min})\theta(H_0 - \overline{H}_{min})dp_x = 0.$$

In the case $k' \equiv \exp$ this condition can never vanish because $\theta(x) > 0$ for all x and $(H_0 - \overline{H}_{min})$ has the same sign for all p_x. So possibilities for k' only include functions that are odd with respect to v_x, but if k attains negative values the full particle distribution function might not remain positive everywhere, e.g. trigonometric k. In conclusion this example works from a mathematical point of view, but not from a physical one.

Alternatively, we can choose both Fourier transforms in condition Eq. (2) directly and then determine g and k. This "inverse" approach is more difficult to apply compared to the one described above because it is more unpredictable whether the resulting pair of functions will make sense physically. As an example we combined Heaviside functions with a Gaussian for $\hat{F}(u) = \theta(u - u_0)e^{-au^2}$ and an arbitrary function h for $\hat{k}'(u) = \theta(u_1 - u)h(u)$ with u_0 and u_1 such that Eq. (2) is satisfied. This choice leads to k and g, and also particle density and pressure tensor contribution of Δf, being defined through exponential functions, Error functions and exponential integrals Ei(x) which either would need to be determined numerically or may not exist at all.

We have introduced a mathematical method that can help to find distribution functions for collisionless force-free current sheets that lead to asymmetries in density and temperature. Using a separable representation and the Fourier transformation we have confirmed why examples listed in Neukirch et al. (2020) [1] work and have found examples of distribution functions that are mathematically correct but do not fulfil all physical requirements such as integrability over velocity space. In some cases one could in future use numerical methods to solve integral involving special functions.

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