Impact of Suprathermal and Beam Electrons on Nonlinear Electrostatic Waves in an Electron-Positron Plasma

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Abstract. The propagation of electrostatic acoustic waves is considered in an electron-positron plasma consisting of inertial electrons, beam electrons, inertial positrons, and suprathermal hot electrons. We see that suprathermality and the physical conditions of beam electrons and positrons affect the nonlinear properties and existence domain of electrostatic solitray waves.

1. Introduction. Electrostatic waves are commonly generated in high-energy astrophysical plasmas such as pulsars and microquasars having electron populations with distinct temperatures [1], which may also contain a fraction of magnetic field-aligned beam electrons [2] and positrons [3], while hot electrons seem to follow a suprathermal κ-distribution [4]. We therefore consider the nonlinear formation of electrostatic acoustic solitary waves in an electron-positron plasma that consists of an electron fluid, a suprathermal hot electron background, an electron beam, and a positron fluid. We look at how the superthermality of electrons and the physical conditions of positrons [5] and electron beams [6] change the nonlinear properties and existence domain of electrostatic solitary waves.

2. Fluid Model. We consider a collisionless plasma model consisting of inertial electrons ('e'), inertial positrons ('p'), and beam electrons ('b') described by the following fluid equations:

$$\frac{\partial n_e}{\partial t}+\frac{\partial (n_e u_e)}{\partial x}=0, \qquad \frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}=\frac{e}{m_e}\frac{\partial \phi}{\partial x}, \tag{1}$$

$$\frac{\partial n_p}{\partial t}+\frac{\partial (n_p u_p)}{\partial x}=0, \qquad \frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}=-\frac{e}{m_e}\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial (n_b u_b)}{\partial x}=0, \qquad \frac{\partial u_b}{\partial t}+u_b\frac{\partial u_b}{\partial x}=\frac{e}{m_e}\frac{\partial \phi}{\partial x}, \tag{3}$$

which are coupled via Poisson's equation to stationary ions and hot suprathermal electrons described by a κ-distribution function:

$$\frac{\partial^2 \phi}{\partial x^2}=-(1+\alpha+\beta-\gamma)+n+\beta n_b-\gamma n_p+\alpha\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+\frac{1}{2}}. \tag{4}$$

We normalized all quantities as follows: electron density $n \doteq n_e/n_{e,0}$, electron velocity $u \doteq u_e/c_{th}$, positron density $n_p \doteq n_p/n_{p,0}$, positron velocity $u_p \doteq u_p/c_{th}$, beam density $n_b \doteq n_b/n_{b,0}$, beam velocity $u_b \doteq u_b/c_{th}$, electric potential $\phi \doteq \phi/(k_B T_h/e)$, time $t \doteq t\,\omega_{pc}$, and space $x \doteq x/\lambda_0$; while the parameters are define by $\alpha \equiv n_{h,0}/n_{e,0}$, $\gamma \equiv n_{p,0}/n_{e,0}$, $\beta \equiv n_{b,0}/n_{e,0}$, equilibrium beam

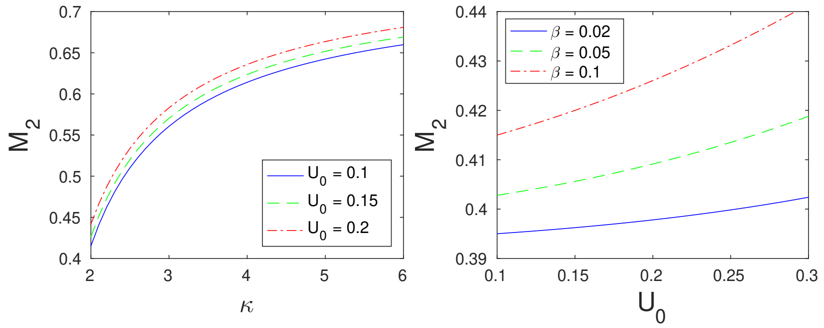

Figure 1: The upper limit for the Mach number (M_2) of positive potential solitary waves as a function of κ for different U_0 (left panel), and a function of U_0 for different β (right panel). The remaining parameters are $\kappa = 2$, $\alpha = 1.2$, $\beta = 0.1$, $\gamma = 0.01$, unless specified.

speed $U_0 \equiv u_{b,0}/c_{th}$, $c_{th}^2 \equiv k_B T_h/m_e$, $\omega_{pc}^2 \equiv n_{e,0}e^2/\varepsilon_0 m_e$, and $\lambda_0^2 \equiv \varepsilon_0 k_B T_h/n_{e,0}e^2$, where $n_{i,0}$ is the unperturbed density of i species, T_h is the hot electron temperature, k_B the Boltzmann constant, m_e the electron mass, and ε_0 the permittivity constant.

3. Linear and Nonlinear Waves. Linearizing Eqs. (1)–(3), substituting into Poisson's equation, and restricting up to the first order result in the following *linear dispersion relation*:

$$\omega^2 = \frac{k^2}{k^2 + \alpha\left(\frac{\kappa-1/2}{\kappa-3/2}\right)}\left[1+\gamma+\frac{\beta}{(1-kU_0\,/\omega)^2}\right]. \tag{5}$$

Without suprathermal and beam electrons, it reduces a dispersion relation derived by Ref. [7].

Transforming Eqs. (1)–(3) into a stationary frame traveling at a speed (Mach number) M, so $\xi = x - Mt$, resulting in $u = M(1-1/n)$, $u = M - \left(M^2+2\phi\right)^{1/2}$, $u_p = M(1-1/n_p)$, $u_p = M - \left(M^2-2\phi\right)^{1/2}$, $u_b = M[1-1/n_b(1-U_0/M)]$ and $u_b = M - \left(M^2+2\phi-2MU_0+U_0^2\right)^{1/2}$. Substituting them into Poisson's equation and then integrating yield a *nonlinear pseudo-energy balance equation*:

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \Psi(\phi) = 0, \tag{6}$$

with the "Sagdeev" pseudopotential $\Psi(\phi)$ defined by

$$\begin{aligned}\Psi(\phi) =& (1+\alpha+\beta-\gamma)\phi + M^2\left[1-\left(1+\frac{2\phi}{M^2}\right)^{1/2}\right] - \gamma M^2\left[1-\left(1-\frac{2\phi}{M^2}\right)^{1/2}\right] \\ &+ \beta(M-U_0\,)^2\left[1-\left(1+\frac{2\phi}{(M-U_0\,)^2}\right)^{1/2}\right] + \alpha\left[1-\left(1-\frac{\phi}{(\kappa-\frac{3}{2})}\right)^{-\kappa+3/2}\right].\end{aligned} \tag{7}$$

Soliton existence domain. To propagate solitary waves, we should impose $F_1(M) = -\Psi''(\phi)|_{\phi=0} > 0$ that leads to a lower limit for the Mach number $F_1(M) = \frac{\alpha(\kappa-\frac{1}{2})}{\kappa-\frac{3}{2}} - \frac{1}{M^2} - \frac{\gamma}{M^2} - \frac{\beta}{(M-U_0\,)^2}$. Moreover, an upper limit for the Mach number is obtained by $F_2(M) = \Psi(\phi)|_{\phi=\phi_{\max}} > 0$, so reality of the density variables correspond to the negative potential $\phi_{\max(-)} = -M^2/2$ for $U_0 \leq 0$ and

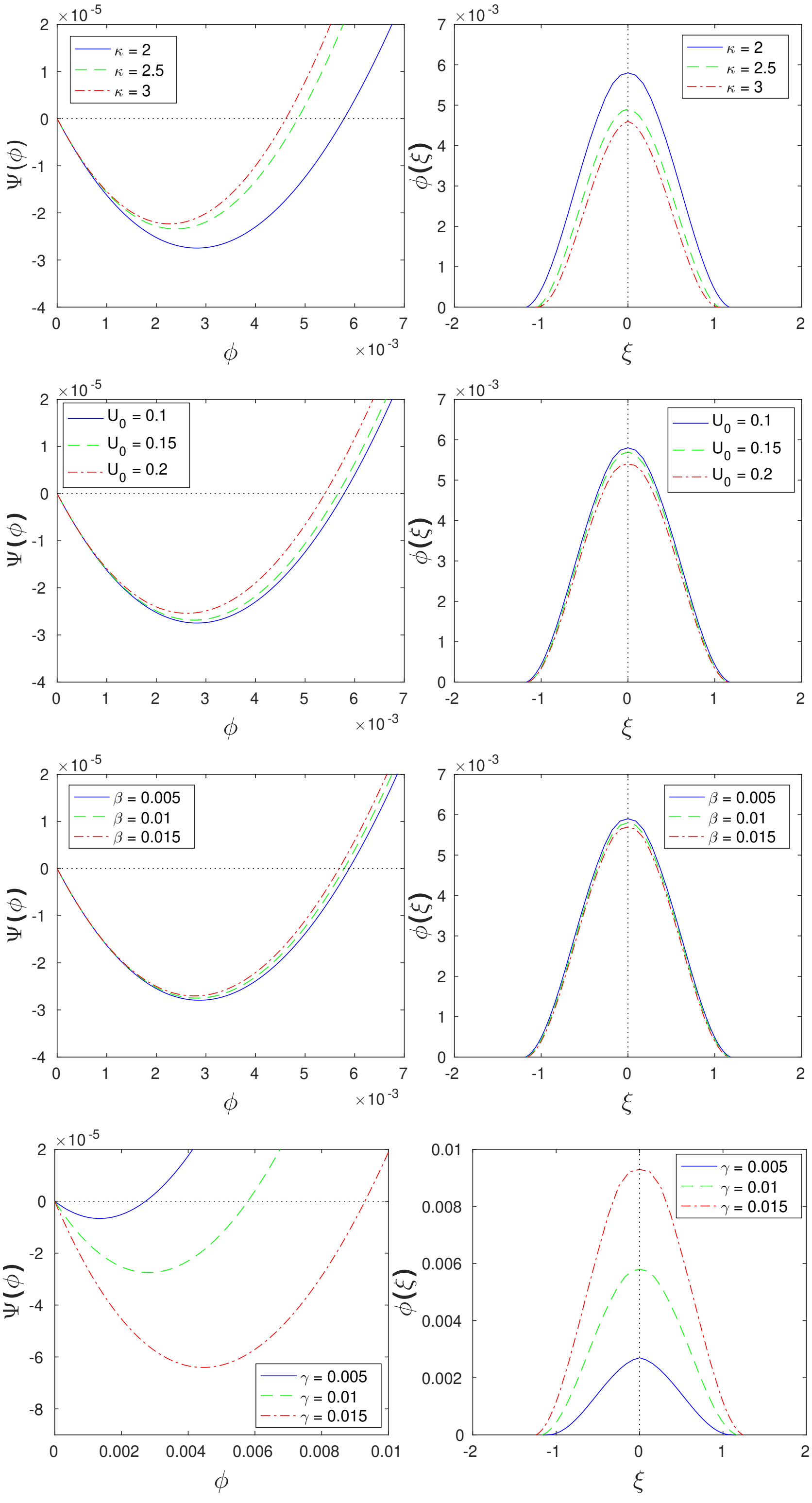

Figure 2: Variation of the pseudopotential $\Psi(\phi)$ with the electrostatic potential ϕ (left panels) and ϕ with position ξ (right panels) for different values of κ (1st row; where $\alpha = 1.2$, $\beta = \gamma = 0.01$, $U_0 = 0.1$, $M = 0.3$), beam speed U_0 (2nd; $\kappa = 2$, $\alpha = 1.2$, $\gamma = 0.01$, $M = 0.3$), the beam-to-electron density ratio β (3rd; $\kappa = 2$, $\alpha = 1.2$, $\gamma = 0.01$, $M = 0.3$), and the positron-to-electron density ratio γ (4th row; $\kappa = 2$, $\alpha = 1.2$, $U_0 = 0.1$, $\beta = 0.01$, $M = 0.3$).

$-(M-U_0)^2/2$ for $U_0>0$, and the positive potential $\phi_{\max(+)}=M^2/2$. We obtain for negative potential solitons ($\phi\leq 0$) and $U_0>0$:

$$F_2(M)=-\tfrac{1}{2}(1+\alpha-\beta-\gamma)(M-U_0)^2+M^2\left[1-\left(1-\frac{(M-U_0)^2}{M^2}\right)^{1/2}\right]$$
$$-\gamma M^2\left[1-\left(1+\frac{(M-U_0)^2}{M^2}\right)^{1/2}\right]+\alpha\left[1-\left(1+\frac{(M-U_0)^2}{2(\kappa-\frac{3}{2})}\right)^{-\kappa+3/2}\right]. \quad (8)$$

and for positive potential solitons ($\phi>0$):

$$F_2(M)=\frac{1}{2}\left(1+2\left(1-\sqrt{2}\right)+\alpha+\beta-\frac{3}{2}\gamma\right)M^2+\alpha\left[1-\left(1-\frac{M^2}{2(\kappa-\frac{3}{2})}\right)^{-\kappa+3/2}\right]$$
$$+\beta(M-U_0)^2\left[1-\left(1+\frac{M^2}{(M-U_0)^2}\right)^{1/2}\right]. \quad (9)$$

The lower limit (M_1) for solitary waves is constrained by $F_1(M)$ and $M>U_0$, whereas the upper limit (M_2) is provided by $F_2(M)$. The solitary waves propagate between M_1 and M_2 (see Fig. 1).

4. Parametric Effects. To study the properties of electrostatic solitary waves, we employed a numerical integration method to solve Eqs. (6) and (7) (under the condition $\beta U_0 \ll 1$ [8]), which help us investigate parametric effects on the wave structures. As seen in Fig. 2, the positive amplitudes increase with strong superthermality (low κ) in hot electrons. The amplitudes of positive solitons decline with increases in the beam speed and density. The positive electrostatic amplitude rises with increasing the positron density relative to the inertial electron density.

5. Summary. The existence domain of positive potential solitary waves shrinks with increases in superthermality (reducing κ; see Fig. 1) and the positron density, and decreases in the beam speed and density. Stronger superthermality (lower κ), higher positron densities, and lower beam densities lead to higher positive potential soliton amplitudes at a fixed Mach number as seen in Fig. 2. Our results improve our understanding of electrostatic solitary waves in electron-positron astrophysical plasmas, where field-aligned beams and suprathermal electrons exist.

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