Spectral features and energy cascade of plasma turbulence at sub-ion scales

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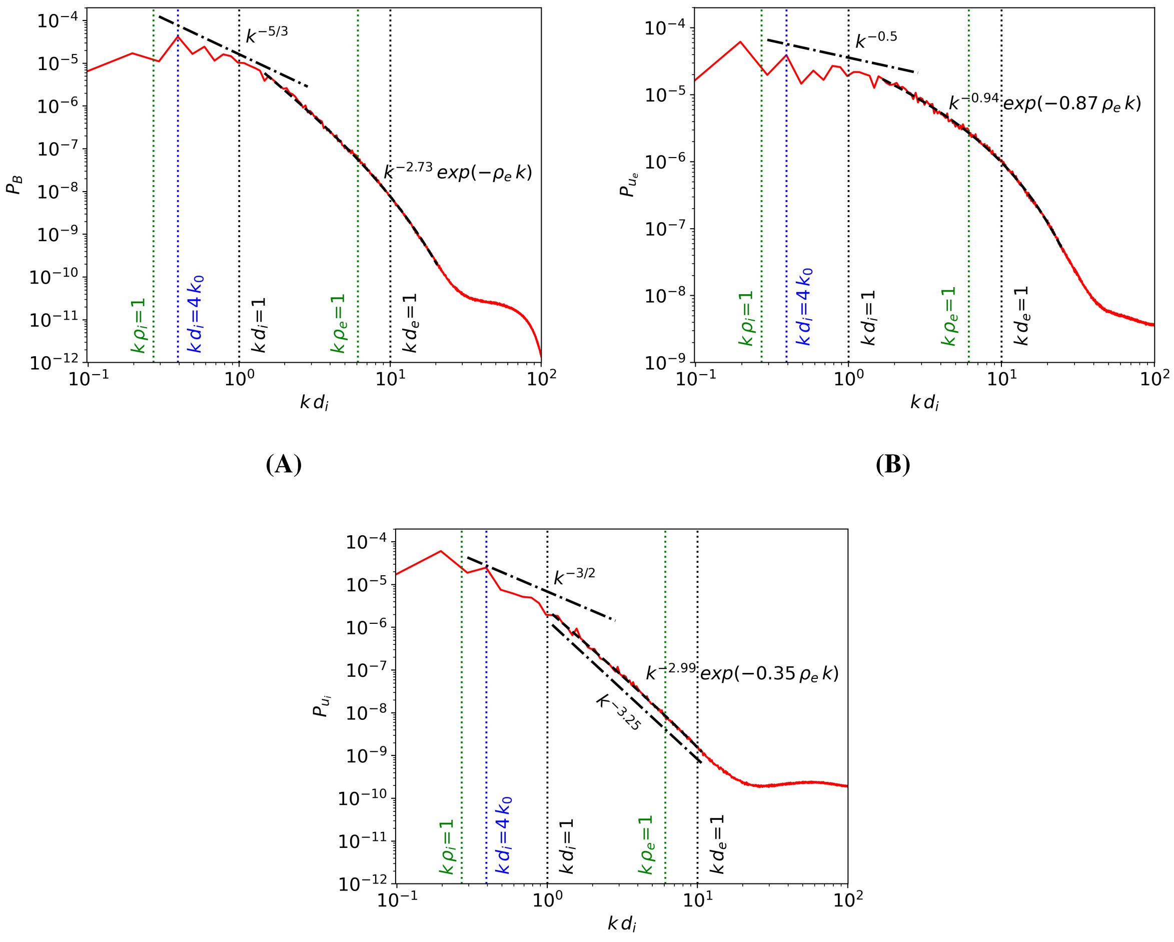
Plasma turbulence is a complex process whose multiscale nature is manifested in the properties of the turbulent spectra that exhibit different behavior and shapes in different ranges of scales. Satellite measurements in the solar wind (SW) and in the Earth's magnetosheath clearly show that at magnetohydrodynamic (MHD) scales the magnetic field spectrum typically follows a Kolmogorov-like power law $\sim k^{-5/3}$ that breaks and develops into a steeper power law at ion scales [Alexandrova et al., 2013]. Recent observations have also revealed the presence of a second break at electron scales, where the magnetic spectrum becomes even steeper. Since clear electron scale measurements are very difficult to obtain, the shape of the magnetic spectrum in this range is still under debate and it is not clear if a new power law develops or if the spectrum decays exponentially [Alexandrova et al., 2012, 2021, Sahraoui et al., 2013]. Moreover, a related problem is understanding the dynamics responsible for the steepening observed at ion and electron kinetic scales.

In our work we analyze the spectral properties of plasma turbulence at sub-ion scales by means of a fully kinetic simulation of freely decaying turbulence realized with the energy conserving particle-in-cell code ECsim [Markidis and Lapenta, 2011]. We use a square periodic box of size $L = 64\,d_i$ (where d_i is the ion inertial length), with 2048^2 cells and 5000 particles per cell for both ions and electrons. The mass ratio is $m_i/m_e = 100$, corresponding to an electron inertial length $d_e = 0.1\,d_i$. Both species are initialized from an isotropic Maxwellian with uniform density and temperature, characterized by a plasma beta equal to $\beta_i = 8$ for the ions and $\beta_e = 2$ for the electrons. With these betas, the ion and electron gyroradii are respectively $\rho_i = \sqrt{\beta_i}\,d_i \simeq 2.83\,d_i$ and $\rho_e = \sqrt{\beta_e}\,d_e \simeq 1.41\,d_e$. An out-of-plane guide field is present and the turbulence is triggered by random magnetic field and velocity perturbations with wavenumber k in the range $1 \leqslant k/k_0 \leqslant 4$ (where $k_0 = 2\pi/L$). The simulation is advanced with a time step $\Delta t = 0.05\,\Omega_e^{-1}$ (where Ω_e is the electron cyclotron frequency).

Fig. 1 shows the magnetic field, electron velocity and ion velocity spectra at fully developed turbulence. We fit these spectra using the *exp* model $k^{-\alpha}\,exp(-\lambda\,k)$ introduced in Alexandrova et al. [2012] to describe the shape of the magnetic field spectrum observed in the SW at sub-ion scales. Each fit starts around $k\,d_i \simeq 1.5$, where the ion scale break is observed, and stops at the inflection point where the spectrum becomes convex because of numerical noise. We see that the magnetic field and electron velocity spectra are well described by the *exp* model, with an exponential factor that becomes important at scales of the order of ρ_e. On the other hand, the ion velocity spectrum does not show a strong exponential behavior and its shape is more consistent with a steep power law $\sim k^{-3.25}$, where the scaling exponent -3.25 was obtained by fitting the spectrum with a pure power law model.



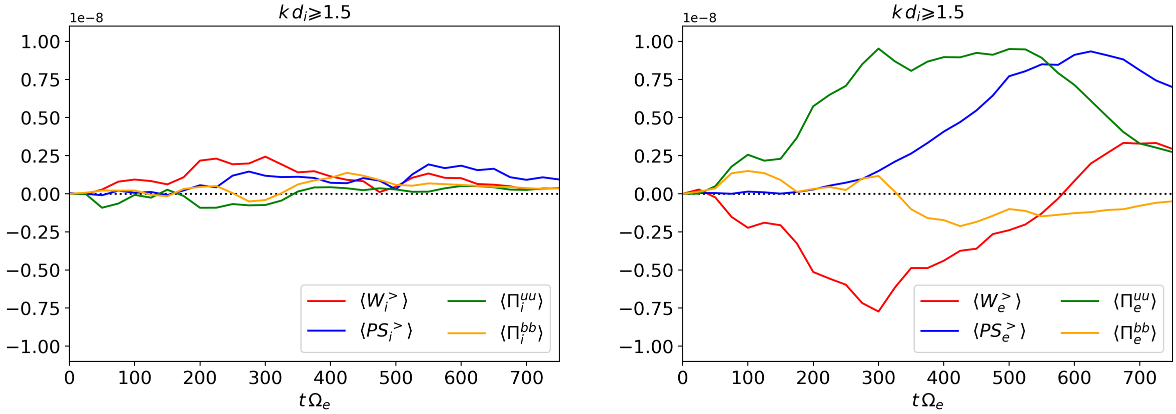
(A)

(B)

(C)

Figure 1: (A) Magnetic field spectrum, (B) electron velocity spectrum and (C) ion velocity spectrum at $t = 650\,\Omega_e^{-1}$.

The presence of the electron scale exponential falloff in both the magnetic field and electron velocity spectra (which is instead absent in the ion velocity spectrum) suggest that the electrons may play a dominant role with respect to the ions in guiding the magnetic field dynamics at sub-ion scales. To investigate this problem we analyze the filtered energy conversion channels introduced in Matthaeus et al. [2020] to study the energy exchanges between different ranges of scales. In particular, we consider the high-pass filtered electromagnetic (e.m.) work $W_s^>$, the high-pass filtered pressure-strain interaction $PS_s^>$ (accounting for the conversion of fluid flow energy into internal energy) and the cross-scale fluxes Π_s^{bb} and Π_s^{uu}, respectively describing the transfer of e.m. energy and of fluid flow energy from large to small scales (the subscript $s = i, e$ indicates the species). The high-pass filtered energy conversion channels are obtained as the difference between the corresponding unfiltered channels and the low-pass filtered ones (the definition of the low-pass filtered channels and of the cross-scale fluxes can be found in Matthaeus et al. [2020]).



(A) **(B)**

Figure 2: Time evolution of box-averaged high-pass filtered energy conversion channels of (A) ions and (B) electrons at scales $kd_i \geqslant 1.5$.

Fig. 2 shows the time evolution of the box-averaged energy conversion channels at scales $kd_i \geqslant 1.5$ (where $\langle \cdot \rangle$ indicates the box-averaging). We see that the ion channels are less efficient than the electrons ones. $\langle \Pi_i^{bb} \rangle$ and $\langle \Pi_i^{uu} \rangle$ fluctuate around zero during the first half of the simulation and become slightly positive for $t > 330\,\Omega_e^{-1}$, indicating that not much energy is transferred from large to sub-ion scales by the ions. $\langle W_i^> \rangle$ is positive during the whole simulation, meaning that the ions are constantly taking energy from the e.m. field at sub-ion scales. Part of this energy is dissipated by the $\langle PS_i^> \rangle$ that is also always positive. On the other hand, $\langle \Pi_e^{uu} \rangle$ indicates that the electrons are transporting a large amount of fluid flow energy from large to sub-ion scales. This energy is partially transferred to the e.m. field by $\langle W_e^> \rangle$ that is negative for $t < 570\,\Omega_e^{-1}$. This means that the electrons are supporting the sub-ion scale magnetic field dynamics since they are giving energy to the e.m. field, differently from the ions that are instead taking energy from it. The electron $\langle PS_e^> \rangle$ is positive and rapidly increases from the beginning of the simulation. At about $t \simeq 570\,\Omega_e^{-1}$ we see that $\langle PS_e^> \rangle$ become larger than $\langle \Pi_e^{uu} \rangle$, which means that dissipation becomes more efficient than the cross-scale flux of energy. At this point the electrons are at a loss for energy (since more energy is dissipated than the amount coming from large scales) and they start to take energy back from the magnetic field, making $\langle W_e^> \rangle$ become positive. This implies that for $t > 570\,\Omega_e^{-1}$ the electrons stop supporting the magnetic field dynamics and start to dissipate the e.m. energy by converting it first into fluid flow energy via $\langle W_e^> \rangle$ and finally into internal energy through the pressure-strain interaction $\langle PS_e^> \rangle$.

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