FEQIS: A new equilibrium solver for the Fenix flight simulator

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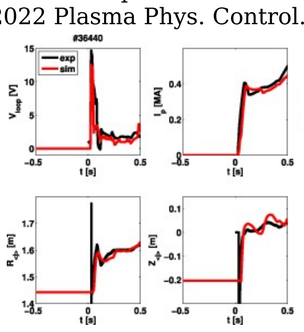
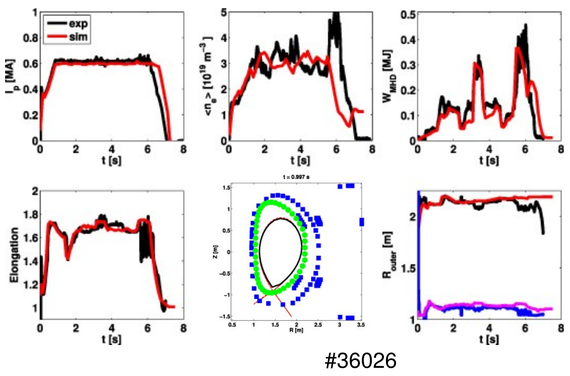
KEY POINTS:

- **Fenix tokamak flight simulator [1,2]: predicts a full discharge including the behavior of the plasma and the control system**
- **The plasma model is provided by the ASTRA transport code [3,4].**
- **A new equilibrium solver: FEQIS, has been written to be coupled to ASTRA.**
- **FEQIS is optimized for fast free-boundary simulations including the circuit equations with flexible description of the circuit connections.**

Preposition: solve the full-discharge evolution

- The goal of Fenix is to predict a full discharge starting from the vacuum evolution (charging of the OH current and of the poloidal field coils), to the plasma flat-top, back to the ramp-down and plasma termination
- Some key pre-requisites & known issues:
  - The simulator has to be fast (max 1-3 minutes for 10 s of discharge) → needs fast models
  - The Grad-Shafranov equation is a non-linear equation [5] → requires an iteration loop → slow
  - Resolution is an issue → trade-off between speed and accuracy
  - The circuit equations can have arbitrary complexity → connections can also be time-dependent! (e.g. sudden insertion of a resistor switch or a bifurcation path)
  - The transition from no-plasma to plasma should be smooth and physically sensible
- Below an example of the full discharge simulation needed to be addressed, taken from [E. Fable et al., 2022 Plasma Phys. Control. Fusion 64 044002]





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- The plasma itself can have arbitrarily complex shape (e.g. multiple X-points, snowflage, etc.)

Numerical scheme(s)

- Starting point is the Grad-Shafranov equation [5]: $\Delta^* \Psi \propto R\left(j_\phi(R,\Psi)+j_{ext}\right)$ (1)
  - The non-linear dependence requires Picard iterations
  - External currents give a vacuum solution via Green functions method [8]
- The initial equilibrium is obtained using the same stabilization method as described in [6]
- The dynamical evolution of the plasma + circuits is obtained by solving eq. (1) together with the circuit equations:

$$\bar{L}\frac{d\vec{I}}{dt}+\bar{R}\vec{I}=\vec{V}+\frac{d\vec{\Psi}}{dt} \qquad (2)$$

- where L, R are the inductance and resistance matrices, I is the vector of circuit currents, V is the vector of applied voltages, and the last term is the contribution of the flux generated by the plasma inside the coils regions.

- The external loop voltage at the plasma boundary is then computed as: $V_{loop}^{ext}=\sum_{i=1}^{N}\frac{d\Psi_i^{ext}}{dt}$ (3)
  - where the external flux is averaged over the plasma boundary
- The external loop voltage is then used inside the boundary condition for the poloidal flux of the plasma:

$$V_{loop}=\frac{d\Psi_b}{dt}=V_{loop}^{ext}+\frac{d}{dt}\left[\frac{1}{L}\oint\oint\frac{G}{R}\frac{\partial\Psi}{\partial r}dl\,dl'\right] \qquad (4)$$

→ note that equation (4) is solved in its time-integrated form (i.e. for the flux, not the voltage).
- where the last term represents the plasma internal flux, but it in practice a boundary double integral of the edge derivative weighted by the Green function. Equation (4) allows to solve for the edge poloidal flux value and its gradient at each time step. As such, the plasma toroidal current Ip is an output of this calculation.

- The solution of equation (1) is performed on a rectangular grid (R,Z), using the sine transform method in the Z direction.
- The value of the poloidal flux of the plasma boundary is found by comparing existence and flux of X-points to the flux at the limiters. Once this value is given, the new plasma current density is assigned inside the plasma.
  - X (and O)-points are found using a bi-quadratic parametrization on a 9-points local grid, of the type:

$$\Psi(R,Z)=c_1R^2Z^2+c_2R^2Z+c_3RZ^2+c_4RZ+c_5R^2+c_6Z^2+c_7R+c_8Z+c_9 \qquad (5)$$

  - Once the coefficients c_j are found, the 0 of the Psi gradient is simply obtained by local Newton search in 2D.
- After the 2D free-boundary solution is converged, the boundary is passed to a 2D fix-boundary solver (developed as well inside FEQIS), that refines the calculation of the closed flux surfaces and provides the core metric coefficients to ASTRA.

- A few inverse modes available:
  - Find the passive currents Fourier decomposition that best fits a given magnetic axis position (done)
  - Find the passive currents Fourier decomposition that best fits a given boundary (done)
  - Re-fit the coil and passive currents to minimized a functional that includes plasma boundary points, magnetic axis position, and X-point position (in progress).

Speeding up the solver

- The full calculation of 1 single time slice magnetic equilibrium can take from ~ ms to ~ s computational time, depending on the grid resolution, accuracy required on the converged solution, and numerical methods used. For FENIX, the maximum allowed computational time (without any sort of parallelization) should be in the order of ~ ms.
- As such, any reasonable cut in resolution and computations is wellcome. In FEQIS, these cuts have been devised as to not affect the actual physical results qualitatively, nor quantitatively, inside the expected numerical accuracy when working at coarser resolution.
- Below, the several cuts applied are listed and discussed in details.

1) GS equation is directly solved once x time step, including 1 time step advance of the circuit equations. Picard iterations are not carried out → valid of time step is small enough compared to resistive time scale of plasma motion.

2) Grid resolution maintained at 65 x 65, radial/poloidal grid for contouring at 21 x 32.

3) The contouring method is akin as solving the prescribed boundary equation inside the new found boundary, but doing only 5 iterations instead of converging the solution.

Initial convergence

- At the first free boundary call, the static equilibrium is obtained consistently by using the method of artificial currents, that is: the full poloidal flux is written as:

$$\Psi=\Psi_{ext}+\Psi_{plasma}+\tilde{\Psi}_R R^2+\tilde{\Psi}_Z Z \qquad (6)$$

where the two artificial constants Psi_R and Psi_R are numbers.
- These numbers are fitted as to make the magnetic axis sit at a controlled position R_{ctrl},Z_{ctrl}

- Then, the functional relation $\tilde{\Psi}_R(R_{ctrl},Z_{ctrl})$; $\tilde{\Psi}_Z(R_{ctrl},Z_{ctrl})$ is numerically iterated such as to force the two values to 0, using a simple Newton scheme:

$$0=\tilde{\Psi}_R+\delta_R\frac{\partial\tilde{\Psi}_R}{\partial R_{ctrl}}+\delta_Z\frac{\partial\tilde{\Psi}_R}{\partial Z_{ctrl}}$$
$$0=\tilde{\Psi}_Z+\delta_R\frac{\partial\tilde{\Psi}_Z}{\partial R_{ctrl}}+\delta_Z\frac{\partial\tilde{\Psi}_Z}{\partial Z_{ctrl}}$$

; this 2x2 matrix system is solved for the (δ_R, δ_Z) unknowns, and iterated until both values reah below a certain tolerance
→ the derivatives are obtained numerically by solving 3 equilibria with orthogonal displacements of the order of the grid size

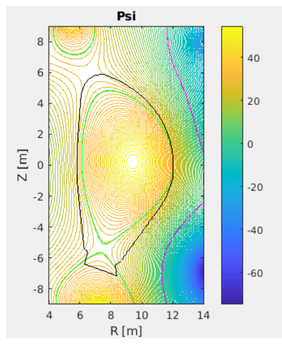
Application to DEMO equilibrium evolution

- To test the new equilibrium solver, we adopt the EU-DEMO scenario described in [7]

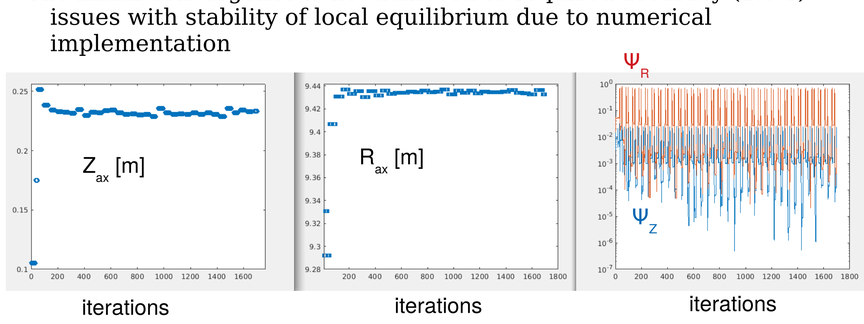
- Here the machine parameters:
  - this scenario assumes an H factor ~ 1

- DEMO parameters:
  - **R = 8.94 m**
  - **a = 2.88 m**
  - **B_T = 4.89 T**
  - **I_p = 19.01 MA**
  - **n_{avg} ~ 8 * 10^{19} m^{-3}**
  - **k ~ 1.7 ; delta ~ 0.4**

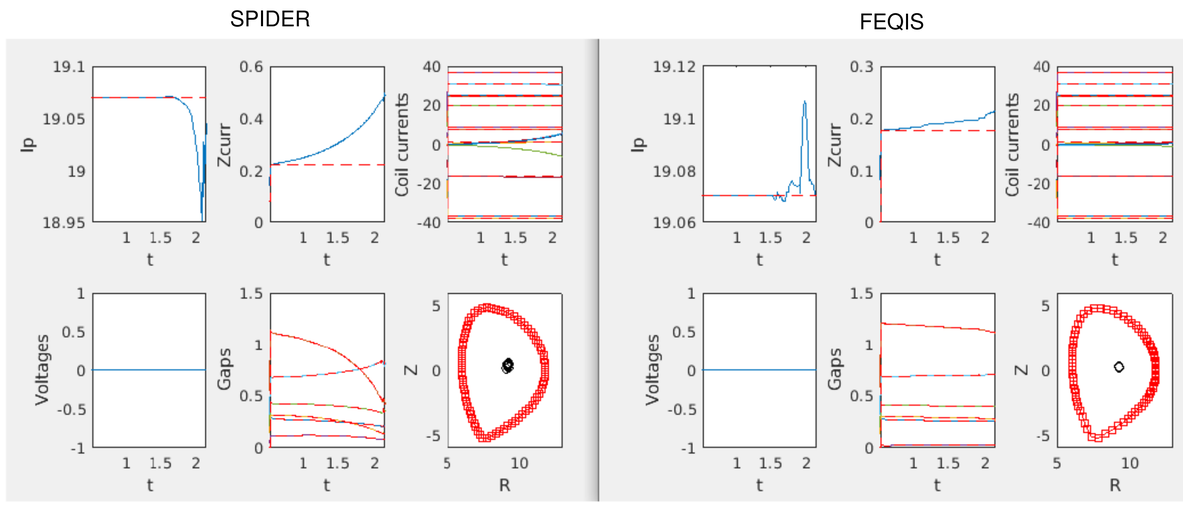
- The initial shape is obtained using coil currents provided by the CREATE group (as well as the coils configuration and parameters)



- The initial convergence is not achieved to required accuracy (1.e-8) → issues with stability of local equilibrium due to numerical implementation



- The numerical issue of not converging monotonically propagates to the time evolution as well, as can be seen in this comparison between FEQIS and SPIDER:



Open issues and future extensions

* OPEN ISSUES

- Still issues in the convergence due to inaccuracies in the solution dependence on the external flux

- The vacuum field evolution has been instead successfully validated against SPIDER

- More accurate calculation of the Green function integrals [8] to be implemented (logarithmic divergence and partial integrals to be implemented analytically)

- Sine transform calculated brute force → adding a sine transform library method later (but improvement in speed is minimal at 65x65 resolution)

* FUTURE DEVELOPMENTS

- Add the toroidal flow term to the equilibrium equation [9]:

$$\nabla^2\Psi\propto FF'+e^{\gamma}P'+\left(flow\,terms\propto\omega^2\right) \quad ; \quad \gamma=\frac{M}{T}\left(R^2-R_0^2\right)\frac{\omega^2}{2} \qquad (7)$$

- centrifugal displacement of pressure iso-lines to the LFS → recovers static GS for omega = 0

- Add the pressure anisotropy term to the equilibrium equation [10]:

$$\nabla\cdot\left[(1-\sigma)\frac{\nabla\Psi}{R^2}\right]\propto FF'/(1-\sigma)+P_{\parallel}' \quad ; \quad \sigma\propto\frac{p_{\parallel}-p_{\perp}}{B^2} \qquad (8)$$

- modifications to magnetic properties of the plasma as well (e.g. relation between BT and radius)

- Allow for variable magnetic permeability of the materials (for example, ferritic inserts as envisaged for ITER and DEMO) [11]:

$$\vec{M}\propto\chi(B)\left[\vec{B}_{ext}-\vec{f}(\vec{M})\right]$$
$$\vec{J}_M=\nabla\times\vec{M} \qquad (9)$$

- where χ is the material magnetic susceptibility and J_M is the new current contribution into the GS equation.

References

[1] F. Janky et al., Fus Eng and Design (2019)
[2] E Fable *et al* 2022 *Plasma Phys. Control. Fusion* 64 044002
[3] G. V. Pereverzev and Yu. P. Yushmanov, IPP report 1992
[4] E Fable et al 2013 Plasma Phys. Control. Fusion 55 124028
[5] Grad H., Rubin H., IAEA p. 190, vol 31 (1958)
[6] Ivanov et al 2005 32nd EPS Conf. On Plasma Physics vol 29C (ECA) P-5.063
[7] G. Federici et al, Fus Eng and Design, 173 112959 (2021)
[8] K. Lackner (1976), Computation of Ideal MHD Equilibria, Computer Physics Communication, 12, 33-44.
[9] L. Guazzotto et al., Physics of Plasmas 11, 604 (2004)
[10] A Evangelias et al., 2016 Plasma Phys. Control. Fusion 58 045022
[11] Shijun Du and Shaohua Wang 1995 Nucl. Fusion 35 359

Acknowledgments

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