# An efficient zero-order analysis of resonant effects on fast ion losses.

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Fast ion (FI) losses may compromise the performance of the fusion reactor by transporting energetic particle bursts on the plasma facing components of the wall. Such losses often occur due the interaction of the fast ion population with MHD modes [1, 2] or as a result of synergetic effects between FI and magnetic perturbations (MPs) and FI and neoclassical tearing modes (NTMs). Numerical simulations have shown that application of MPs can lead to significant fast ion losses in ITER [3] and have highlighted the role of resonances in FI transport [4].

In this work, we propose a zero-order analysis of the fast ion dynamics which aims at providing insight on the effect of resonances on fast ion losses. Our analysis is carried out in the unperturbed phase space of the single particle gyrocenter in a background equilibirum magnetic field. The key contribution is the introduction of the *resonance index*, a quantity which models the susceptibility of the gyrocenter phase space to the effect of specific external perturbations.

The value of the *resonance index* depends on the number of resonant orbits in the vicinity of a phase space point as well as the distance of these resonant orbits from the vessel wall. Conceptually, the result is a function of phase space which quantifies the probability that an elementary volume in phase space may yield escaping orbits in the presence of specific MHD modes and magnetic perturbations by the mechanism of resonance overlap. Finally, we propose a recipe of mapping the *resonance index* to the plane of the scintillator of the FILD detector [5]. We demonstrate a remarkable similarity between the mapped resonance index profile and the FILD detector signal for shot #30810 in ASDEX upgrade.

**Single particle dynamics.**

The strong interaction between fast ions and MHD modes or MPs results to the emergence of chaotic orbits, due to the destruction of Kolmogorov-Arnold-Moser (KAM) curves [6] on the fast ion phase space. In particle phase space, the domains of chaotic motion are centered around the positions where unperturbed orbits are in resonance with the existing perturbations.

The extent of such domains depends on the strength of the perturbations and can be estimated by means of the canonical perturbation theory.

In order to be considered complete, a model of the fast ion interaction with the perturbation would require the calculation of the chaotic phase space domains. However, a preliminary study restricted to the calculation of the resonance positions alone can be illuminating. While the former analysis requires specific knowledge of the perturbation profile and amplitude, quantities that are not always known, the latter needs only information about the perturbation frequencies and wave numbers. Information of the unperturbed resonant orbits alone is enough to yield results that are in qualitative agreement with the experimental data from the FILD detector and reveal the underlying mechanisms that lead to fast ion loss.

Due to the nonlinear nature of the guiding center dynamics, a monochromatic perturbation of the form $\sigma = A(\psi)\exp(i(n\phi + m\theta - \omega t))$, yields an infinite series of harmonics, when expressed in action angle variables, i.e. $\sigma = \sum_l A_{m,l}(J, P_\phi)\exp\left(i\left(m\hat{\phi} + l\hat{\theta} - \omega t\right)\right)$. As a result a monochromatic perturbation with toroidal number $n$ can yield a cascade of resonances at the points where any of the resonance conditions

$$n\omega_\phi + l\omega_\theta - \omega = 0, \quad l \text{ any integer}$$

are met. The amplitude of each harmonic and the extent of the corresponding resonance depends on the profile of the perturbation and on the shape of the unperturbed orbit.

**Methodology**

To begin our analysis, we group orbits into sets that share the same canonical toroidal momentum $P_z$ and the same magnetic moment $\mu$, both of which are adiabatic invariants of the gyrocenter motion. The projection of any such family on the poloidal plane is a set of non intersecting curves, except for the x-points of the separatrices. We will call these families *poloidal slices*.

Each point on the FILD scintillator can be associated with a single orbit that has the same kinematic characteristics (Larmor radius $\rho$ and pitch angle $\theta$). Therefore, each point on the FILD plane corresponds to a single poloidal slice, see fig. 1, so that the resonance index can be mapped onto the FILD plane.

Conceptually, there two major quantities that affect the likelihood for a poloidal slice to yield escaping orbits: the maximum toroidal flux surface (radial position) of the resonant orbits, $\psi_{\text{max}}$ and the number of resonant orbits $n_{\text{res}}$ within the slice. A rough estimate of their combined effect can be given by the *resonance index*:

$$r_i \equiv n_{\text{res}}\psi_{\text{max}}$$

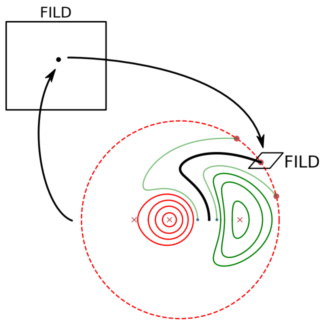


Figure 1: A single trace on the FILD detector corresponds to a poloidal slice.

The resonance index profile provides an indication of ion populations which will no longer be confined. Although its calculation, relies solely on unperturbed orbits quantities, and as such neglects significant physical effects (mode profile, plasma penetration, initial ion distribution and so on), our preliminary analysis indicates that the resonance index profile alone can predict the main characteristics of the trace on the FILD detector. We assert that the FILD trace is the result of the modulation of the resonance index by the ion distribution and the mode profile.

Not all resonances affect the phase space in the same way. In order to have enhanced interaction, the Chirikov criterion dictates that there must be resonance overlap. To account for this, we introduce a free parameter, the *resonance index threshold*. We assume that poloidal slices with resonance index below the threshold do not generate any escaping ions, while the ion flux from poloidal slices above the threshold is proportional to the resonance index.

**Extra modelling steps**

In our analysis, we have approximated the equilibria with an analytic cylindrical model whose q profile is given by $q = q_0 \left[1 + (r/4)^8\right]^{1/4}$. We have chosen this profile, because it is an analytic solution of the Grad Shafravov equation [7] and resembles the profiles of the actual equilibira of the shots.

**Results for AUG #30810.**

In AUG shot #30810 an enhanced ion loss ultimately led to damage of the FILD probe. This loss has been attributed to a strong ion interaction with a $\sim 45kHz$ 5/4 MHD mode [2]. The comparison between the $n_{\mathrm{res}}$ and $r_i$ profiles and the signals obtained from ASCOT and FILDSIM [8, 9] simulations are presented in fig. 2. Notice how the local maxima in fig. 2.b) and fig. 3 coincide, in spite of all the major simplifications, the fact that we have ignored the effect of the resonance strength and the fact that we have made no attempt to account for the initial ion distribution.

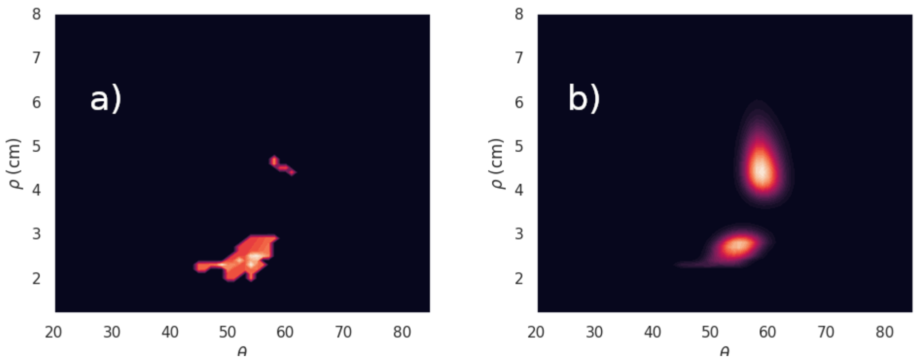


Figure 2: a) The resonace index after threshold taking into account the first 6 poloidal harmonics. b) The resonance index multiplied by the FILD response matrix.

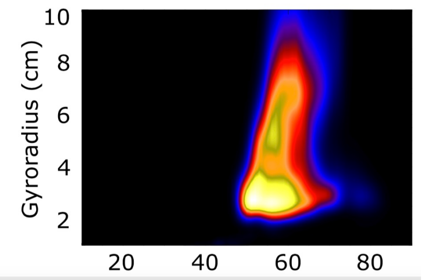


Figure 3: The FILD signal as calculated with ASCOT and FILDSIM, adapted from [2] with permission by the publisher.


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