Resonance broadening in quasilinear theory: towards Kubo >1

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Turbulence is a critical and essential step in understanding and controlling plasmas, like magnetically confined fusion plasmas. In particular, the transport and diffusion of energetic particles from the core to the walls of Fusion reactors. Other domains include particle acceleration driven instabilities from plasma heating [1], magnetic reconnection [2], or laser-produced plasma [3]. Thus, turbulence is an essential part of plasma physics.

In the second half of the 20th century, a revolution in the theory of turbulence started with the introduction of the quasilinear theory (QL) of turbulence [4]. Followed by a correction with the name of resonance broadening (RB) [5].

QL considers nonlinear terms, the evolution of plasma particle distributions outside plasma equilibrium. Physically, it interprets the effects of turbulence as a random variation of particle speed throughout the plasma, resulting in phase-space (PS) diffusion. For a large majority of plasmas, QL theory accurately describes plasma turbulence and particle transport. However, some inconsistencies have been in experiments and simulations [6] [7]. Indeed, nonlinear terms become significant as turbulence increases in amplitude.

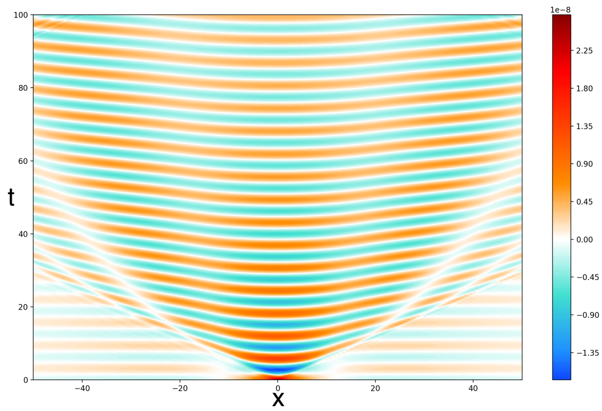

Figure 1: *Arbitrary turbulent electric field autocorrelation function, with Gaussian amplitude.*

The QL limit is expressed in terms Kubo number [8], defined as $K = \tau_0/\tau_b$. Where τ_0 is the turbulent electric field auto-correlation time, and τ_b is the bounce time of electrostatically trapped particles.

Physically, the Kubo number characterizes the type of trajectories charged particles will perform, trapped and free particles. In other words, the Kubo number represents the degree of trapping a turbulent electric field will have over charged particles constituting the plasma.

Indeed, particles evolving in an electric potential/field can get trapped by local potential wells. Typically, a trapped/free particle can not become free/trapped unless the system allows for energy transfers or particle collisions. In the case of plasma, energy transfers are possible in the form of wave-particle resonances, which change the form of the potential well and the energy of particles, leading to transport and diffusion. Hence, quasilinear theory.

Thus, the QL theory describes the plasma in the regime where particle trapping- detrapping is strong $K \ll 1$. In other words, the form of the electric potential changes fast enough for particles to perform partial arcs before changing direction and speed, therefore performing a Brownian-like trajectory in PS. As turbulence amplitude increases $K \ll 1$, particle trapping becomes important. In this case, particle trajectories shift to random-walk-like trajectories for the particle guiding-center, see figure (2).

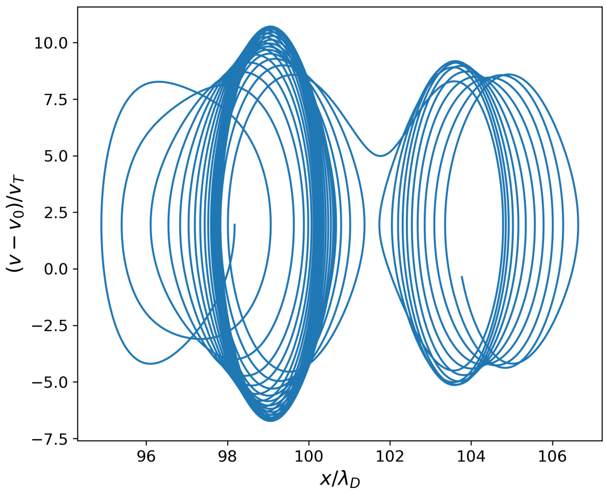

Figure 2: *Phase-space of an arbitrary particle trajectory in a turbulent electric field with Kubo number* $K = 2.24$.

For modern plasma physicists interested in transport and diffusion, the regime of interest becomes $K \geq 1$ [9]. Therefore, a new description of plasma dynamics is required since it is outside the applicability regime of QL theory.

We investigate the turbulent diffusion of particles and compare it against QL theory, including RB. Different diffusion regimes are investigated, $K \ll 1$ and $K \geq 1$ in particular. We discuss the limits of QL theory and possible avenues of study for $K \geq 1$.

Quasilinear theory and diffusion coefitient

As mentioned previously, QL theory is used to compare against diffusion from numerical simulations. QL describes the 1D motion of an ensemble of particles in a prescribed turbulent electric field.

A result of QL and RB theories is an iterative method of calculatin the diffusion coefficient of particles as a function of: Particle velocity v, mass m and charge q, and the autocorrelation function of the electric field $\langle E(0,0)E(x,t)\rangle$ (shown in figure (1)).

$$D^j_{RB}(v) = \left(\frac{q}{m}\right)^2 \int_0^{+\infty} \int_{-\infty}^{+\infty} R_{j-1}(v,x,t)\langle E(0,0)E(x,t)\rangle dxdt \ . \tag{1}$$

where j is the j-th iteration, $R_j(v,x,t)$ is a Gaussian probability distribution of particle velocities which corrects for resonance broadening effects.

The turbulent electric field is expressed as a collection of sinusoidal waves with Gaussian amplitude and Langmuir plasma dispersion relations. Therefore, equation (1) can be simplified by working in the Fourier space. Indeed, since the Fourier transform of a sinusoidal wave and a Gaussian are known functions, this equation becomes

$$D_{RB}^{j}(v) = \left(\frac{q}{m}\right)^2 \int_0^{+\infty} \sum_k \frac{\hat{E}_k^{\,2}}{2} \hat{R}_{k,j} \cos\left[kvt + \omega t\right] dt \ . \qquad (2)$$

Where $\hat{R}_{k,j}$ and $\hat{E}_k$ are the Fourier components of R_j and E respectively.

Numerical model and results

To compare against theory, we study the dynamics of test particles in a prescribed turbulent electric field. In particular, we developed an algorithm that calculates N particle trajectories using a fourth-order Runge-Kutta algorithm.

At $t = 0$, N test particles are initialized with initial random velocities v_0, distributed in a narrow Gaussian probability. And random initial positions. By performing statistics on particle trajectories, we can deduce the diffusion coefficient as the slope of the velocity variance σ_v^2, figure (4) shows an example of velocity variance for an arbitrary simulation in the QL regime.

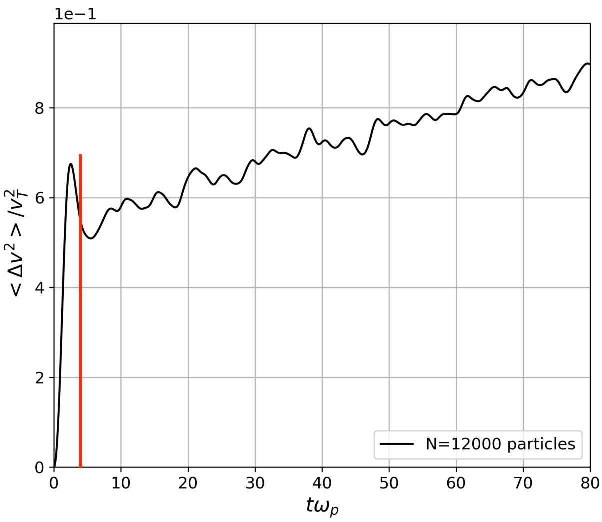

Figure 3: *Velocity standard deviation for $K = 0.07$ and $v_0 = 1.95$ with Gaussian amplitude and Langmuir dispersion. Red line separates the first and second diffusion regimes.*

For all simulations, two regimes of diffusion were observed:

- A first initial regime where velocity variance evolves linearly, whose slope represents the diffusion coefficient.

- The second regime of diffusion, where velocity variance evolves with a slope of less intensity.

Particle diffusion was studied for different values of Kubo number; results are plotted in figure (4). Theoretical curves are calculated using the QL and RB theory, equation (2). We observe qualitative and quantitative agreement between theory and numerical results for $K \ll 1$. Note, the impact of resonance broadening becomes significant for Kubo of the order of a few percent ($K \simeq 0.04$). This impact corresponds to a flattening of diffusion curves, an increase in maximum-diffusion-velocity, and an enlargement of the interval velocities diffuse.

For $K > 1$, we observe qualitative agreement with the theory for velocities around the resonance velocity. However, we measure a non-zero diffusion from numerical simulations for fast particles, while negligible diffusion is predicted by quasilinear theory and resonance broadening.

Moreover, in both diffusion regimes, we observed that the diffusion coefficient evolves as a power of the Kubo number, K^{β}, where β takes different values for different intervals of K. Indeed, for the initial diffusion regime, as Kubo increases, $K > 0.5$ diffusion is proportional to K^{-1}.

Two values of β were observed for the second diffusion regime. For a small Kubo number, $K < 0.2$, the measured diffusion evolves as $K^{-3/2}$. Additionally, around $K = 0.2$, the growth of diffusion shifts to be proportional to $K^{-5/2}$. This shift corresponds to the QL theory limit, which leads to believe that this limit is more fundamental than QL suggests.

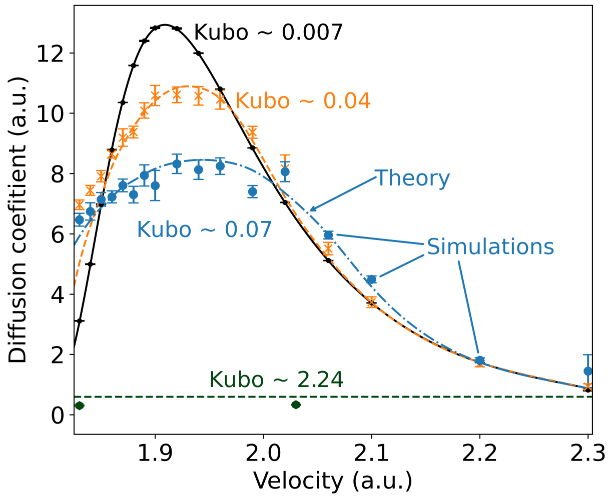

Figure 4: *Quasi-linear diffusion coefficient and numerical diffusion ifor different values of Kubo number, for a random electric field of Gaussian amplitude with Langmuir dispersion.*

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