Linear drive of Fast-ion-driven vertical modes

T. Barberis[1], F. Porcelli[1] and A. Yolbarsop[2]

[1]*Department of Applied Science and Technology, Polytechnic University of Turin, Torino 10129, Italy*

[2]*KTX Laboratory, School of Nuclear Science and Technology, USTC, Hefei, 230022, China*

The vertical instability is a well known instability affecting elongated plasmas, which is associated with axisymmetric perturbations. In tokamak experiments it is stabilized by means of passive feedback consisting of eddy currents induced by the plasma motion in a nearby wall and/or in plasma facing components. The dispersion relation for axisymmetric modes with passive stabilization has been described by [1], and it becomes cubic when wall resistivity is considered [2]. One of the three roots corresponds to a vertical plasma displacement that, in the absence of active feedback stabilization, grows on the wall resistivity time scale. The other two represents oscillatory solutions, with frequency close to the poloidal Alfvén frequency, which are normally damped by wall resistivity. In this work, we discuss a new type of fast particle instability, involving a resonant interaction between fast ions and the oscillatory branch of the axisymmetric mode dispersion relation, as described in [3].

Considering a *straight tokamak* equilibrium with an elliptical cross section and the well known reduced ideal-MHD model, it is possible to use quadratic forms in order to easily derive the cubic dispersion relation for axisymmetric modes, as shown in [4]. The dispersion relation reads:

$$\gamma^3 + \gamma^2 \frac{1}{\tau_\eta} \frac{1}{1-\hat{e}_0 D} + \gamma\omega_0^2 + \omega_0^2 \frac{1}{\tau_\eta} \frac{1}{1-D} = 0 \tag{1}$$

with roots:

$$\omega \approx \pm\omega_0 - i\frac{1}{2\tau_\eta}\frac{D}{(D-1)} = \pm\omega_0 - i\gamma_\eta \tag{2}$$

$$\gamma = \frac{1}{(D-1)\tau_\eta} \tag{3}$$

where the resistive wall time scale τ_η is proportional to the wall resistivity and $\omega_0 \approx e_0^{1/2}\tau_A^{-1}\sqrt{D-1}$ is the mode frequency where e_0 is the ellipticity of the plasma boundary and τ_A the poloidal Alfvén time. D is the so-called geometrical factor, which is related to the relative distance and shape of the wall ensuring passive stabilization. The feedback stabilization of the ideal vertical instability requires $D > 1$, meaning that the two roots of Eq. 2 represents oscillatory solutions damped by wall resistivity.

In order to add the effect of fast ions, we consider the hybrid kinetic MHD model. Within this framework, it is easy to write the general dispersion relation, with fast ions effects, in terms of quadratic forms is

$$-\gamma^2 \delta I = \delta W_{MHD} + \delta W_{fast} \tag{4}$$

where $\delta I = \int_{\mathscr{V}} d^3x \rho \xi \cdot \xi^*/2$, $\delta W_{MHD} = -\int_{\mathscr{V}} d^3x \xi^* \cdot F(\xi)/2$ and $\delta W_{fast} = \int_{\mathscr{V}} d^3x \xi^* \cdot \nabla \cdot \tilde{\mathscr{P}}_h/2$. All the integrals are over the plasma volume $\mathscr{V}$ and ξ is the displacement vector. The fast particles pressure tensor $\tilde{\mathscr{P}}_h$ is defined as moments of the fast ions distribution function and the reduced MHD force operator is $F(\xi) = (\tilde{J} \times B_{eq} + J_{eq} \times \tilde{B})$ with B and J magnetic field and total current density, respectively.

Perturbative approach allows for analytic progress, assuming the energetic particles pressure (or *beta*) much smaller than the core plasma pressure $\beta_h/\beta_c \ll 1$. Therefore all real contribution of δW_{fast} can be safely neglected, while keeping only its imaginary part. As a consequence, the oscillatory branch of the dispersion relation, related to the two roots of Eq.2, modified by fast ions reads:

$$\omega^2 = \omega_0^2 - 2i\omega_0\gamma_\eta + i\omega_0^2\lambda_h + \mathscr{O}(\gamma^2/\omega_0^2) \tag{5}$$

$$\omega = \omega_0 + i\gamma_{tot}, \qquad \gamma_{tot} = \omega_0\lambda_h/2 - \gamma_\eta \tag{6}$$

With the parameter $\lambda_h = Im(\delta\hat{W}_{fast}) \ll 1$, where $\delta\hat{W}_{fast}$ is the normalized fast ion quadratic form. Eq.6 shows that there is a competition between the resistive wall damping and the fast ions resonance drive. After standard manipulations [5], one gets for the relevant part of δW_{fast}:

$$\delta W_h = -\frac{2\pi^2 c}{Zem^2} \sum_\sigma \int dP_\phi d\mathscr{E} d\mu_\perp \tau_\Omega \omega \frac{\partial F}{\partial \mathscr{E}} \sum_{p=-\infty}^{+\infty} \frac{|\Upsilon_p|^2}{\omega + p\omega_\Omega} \tag{7}$$

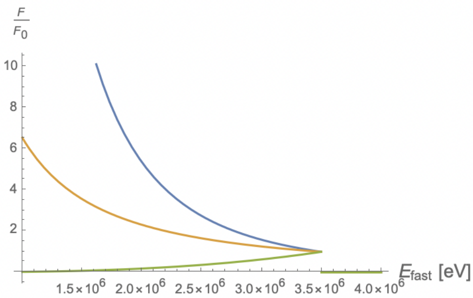
where the integration is over the three constants of fast ions motion, i.e. canonical momentum P_ϕ, energy $\mathscr{E}$ and magnetic moment $\mu_\perp$. The resonant denominator $\omega + p\omega_\Omega$ in Eq.7, with ω_Ω bounce or transit frequency of magnetically confined fast ions, introduces the imaginary part of δW_h. The integer p labels the different harmonics in the particles orbit periodicity and the Fourier coefficients Υ_p describe the contribution to the resonance of each harmonic. After the integration around the pole given by the resonant denominator and normalization, the imaginary part of δW_h, i.e. the parameter λ_h, reads:

$$\lambda_h = C_0 \sum_{p=-\infty}^{+\infty} \int drd\Lambda r \frac{(v_p^*)^3}{h_\Omega} \frac{\partial F}{\partial \mathscr{E}}\bigg|_{v=v_p^*} \left|\Upsilon_p\right|^2 \bigg|_{v=v_p^*} \tag{8}$$

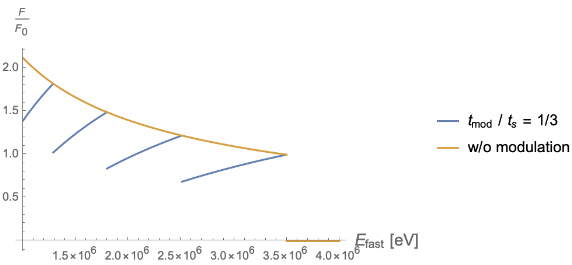
Where v_p^* is the resonant velocity. The sign of λ_h depends solely on $\partial F/\partial \mathscr{E}$, and in particular $\partial F/\partial \mathscr{E} > 0$ is required in order to drive the mode unstable. Such a distribution function can be obtained due to losses of fast ions [6]. Considering a constant monochromatic source and a constant loss frequency ν_{loss}, Fokker-Plank calculations give:

$$f_h(v) = C \cdot \frac{H(v_{fast} - v)}{(v^3 + v_c^3)^{1-\alpha}} \tag{9}$$

with $\alpha = \nu_{loss}\tau_s/3$, τ_s being the slowing down time of fast ions. When the loss term is large enough and $\alpha > 1$, the fast ions cannot relax to the usual slowing down distribution function, and a situation with $\partial F/\partial \mathscr{E} > 0$ is achieved, as shown in Fig.1a



(a) Distribution function for different values of the loss frequency



(b) Distribution function with and without source modulation with $\alpha = 0.8$ at time $t = 6 * t_{mod}$

Figure 1: Plots of normalized distribution function showing the effects of losses and source modulation

As reported by recent experiments [7, 8], a distribution function featuring a positive derivative in the velocity space can also be obtained with a modulation of the fast ion source. In particular, the characteristic time of this modulation (t_{mod}) must be shorter than the slowing down time of the fast ions in order to obtain a $\partial F/\partial \mathscr{E} > 0$. One mechanism that could provide this kind of source modulation for fusion born alphas are sawtooth oscillations, as discussed in [8], due to the drop of the core temperature following a crash. Considering simple models for q and pressure profiles and the Kadomtsev's reconnection model [9], an analytic time-dependent distribution function of fast ions including source modulation effects has been derived. As shown in figure 1b, the modulation of the source due to sawtooth oscillations with period smaller than the slowing down time gives a distribution function with positive $\partial F/\partial \mathscr{E}$, apart from jumps related to the crashes.

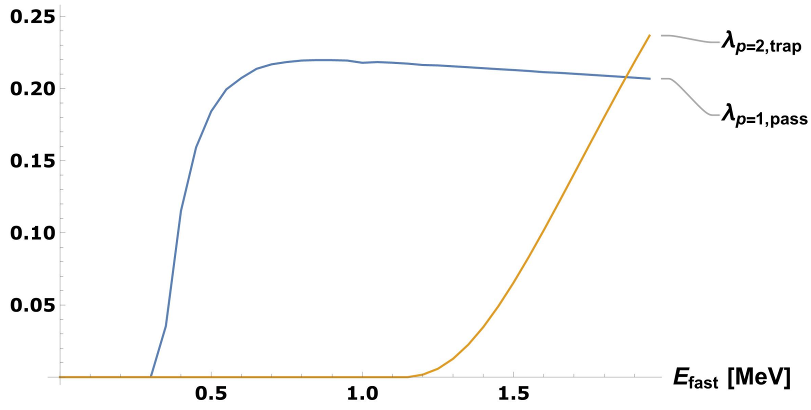


Figure 2: Main contributions to λ' for typical JET plasma parameters and Deuterium fast ions population

For simplicity here we focus on the case with the distribution function of Eq. 9 with $\alpha > 1$. Rewriting $\lambda_h = \lambda'(n_h/n_c) \to (n_h/n_c)_{crit} = 2\gamma_\eta/(\lambda'\omega_0)$, one obtains a stability threshold for the mode in terms of the density ratio between fast and core plasma particles, n_h/n_c. The numerical factor λ' can be subdivided in order to highlight the contributions coming from trapped and passing particles and from different harmonics: $\lambda' = \sum_{p=1,2}\sum_{\Omega=b,t}\lambda'_{p,\Omega}$. Considering typical JET plasma parameters and a Deuterium fast ion population, the main contributions to λ' are displayed in Fig.2. The resulting critical value for the stability threshold is of the order of $n_h/n_c|_{crit} \sim\ 1\times 10^{-2}$.

This theoretical model discussing fast-ion-driven vertical modes (in brief, FIDVM) is partly motivated by the observation of $n = 0$ modes in recent JET experiments [8, 10], interpreted in terms of Global Alfvén Eigenmode (GAE). Estimates of the theoretical mode frequency and stability threshold compares favourably with these experiments.
A more detailed comparison between theory and experiments requires further development of this theoretical framework as well as numerical work.

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