Application of FIDA as a charge exchange loss measurement for NB-produced fast ions in small or medium-size devices

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In NBI-heating experiments in small or medium-size devices such as Heliotron-J and CFQS, the charge exchange (CX) loss of NB-produced fast ions is not negligible in the determination of the fast ions slowing down velocity distribution function. Although it may be possible for many numerical simulation methods for the fast ions to include this loss mechanism, experimental measurements of the neutral particle density profile in the 3-dimmensional real space as the input to such calculations are almost impossible. On the other hand, the FIDA (Fast Ion D-alpha) measurements[1] are now widely used in various experimental devices for investigating the local fast ion velocity distribution. In situations where the CX loss is not negligible and the neutral particle density profile is unknown, this method is not useful for the purpose of the experimental validation of theoretical calculations of the velocity distribution. For the studies of beam-driven neoclassical phenomena such as that in Refs.2-3, and the anisotropic pressure MHD equilibrium states mentioned in Ref.4, however, the requirement on the slowing down velocity distribution is not in the detailed understanding on the slowing down process including the CX loss but in the energy space reduction factor of the lower Legendre order structures of the velocity distribution. As shown below, in both of the direct solving method using the eigenfunction[3] and the indirect solving method based on the adjoint equation method[4], it can be shown that for this purpose that the effect of the CX loss on the velocity distribution will not appear in the pitch-angle space structure but will appear only in the energy space structure. When we find the substantial neutral particle density by comparison of the FIDASIM[1] calculation including this energy space reduction factor and the experimentally observed Balmer-alpha spectrum, we should include this reduction factor also in the calculations of the beam-driven neoclassical phenomena[2], 3, the analyses of the anisotropic pressure MHD equilibriums states[4], and heating power calculations.

Firstly, we shall consider how the formulas in Ref.3 for the flux-surface-averaged 1st Legendre order component are modified by the CX loss term. When the CX loss term to the Coulomb collision operator for the fast ions

$$\begin{gathered}\sum_a C_{\mathrm{f}a}(f_{\mathrm{f}}, f_a) \cong \sum_{a\neq \mathrm{f}} C_{\mathrm{f}a}(f_{\mathrm{f}}, f_{a\mathrm{M}}) \\ \cong \frac{1}{\tau_{\mathrm{S}}}\left[\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2 v_{\mathrm{Te}}\frac{3\sqrt{\pi}}{2}G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3\right\} f_{\mathrm{f}} + \frac{Z_2 v_{\mathrm{c}}^3}{v^3}\mathscr{L} f_{\mathrm{f}}\right] \equiv C_{\mathrm{f}} f_{\mathrm{f}}, \\ \mathscr{L} \equiv \frac{1}{2}\left(\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial}{\partial \xi} + \frac{1}{1-\xi^2}\frac{\partial^2}{\partial \phi^2}\right),\end{gathered} \tag{1}$$

the expansion form using the eigenfunctions for the circulating pitch-angle $0 \le \lambda \le 1$ for $\lambda \equiv (1-\xi^2)B_{\mathrm{M}}/B$ is as following. Although it is not guaranteed that this time constant is surface-quantity, here we assume that it is surface quantity for investigating a surface-averaged effect for this pitch-angle range that makes the dominant contribution to the and the that will discussed below in the tangential NB injections in the Heliotron-J and the CFQS. The solubility condition

$$\frac{1}{2}\left\langle \frac{B}{B_{\mathrm{M}}}\frac{v}{v_{\parallel}}\right\rangle \left[\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2 v_{\mathrm{Te}}\frac{3\sqrt{\pi}}{2}G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3\right\} f_{\mathrm{f}} - \frac{\tau_{\mathrm{S}}}{\tau_{\mathrm{cx}}(v)} f_{\mathrm{f}}\right] + \frac{Z_2 v_{\mathrm{c}}^3}{v^3}\frac{\partial}{\partial \lambda}\lambda\left\langle \frac{v_{\parallel}}{v}\right\rangle \frac{\partial f_{\mathrm{f}}}{\partial \lambda} = 0 \tag{2}$$

for this pitch-angle range expressed by using the eigenfunctions that is defined by

$$2\left\langle \frac{B}{B_{\mathrm{M}}}\frac{v}{v_{\parallel}}\right\rangle^{-1} \frac{\partial}{\partial \lambda}\lambda\left\langle \frac{v_{\parallel}}{v}\right\rangle \frac{\partial \Lambda_n}{\partial \lambda} = -\kappa_n \Lambda_n \mathrm{in} 0 \le \lambda \le 1, \Lambda_n(0) = 1, \Lambda_n(1) = 0 \tag{3}$$

is

$$\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2 v_{\mathrm{Te}}\frac{3\sqrt{\pi}}{2}G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3\right\} f_n(v) - \frac{\tau_{\mathrm{S}}}{\tau_{\mathrm{cx}}(v)} f_n(v) - \kappa_n Z_2 \frac{v_{\mathrm{c}}^3}{v^3} f_n(v) = 0. \tag{4}$$

for $0 \le v < v_{\mathrm{b}}$. Together with the function

$$\ln\mathscr{V}(v) \equiv 3v_{\mathrm{c}}^3 \int \frac{\mathrm{d}v}{v\left\{v^2 v_{\mathrm{Te}}(3\sqrt{\pi}/2)G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3\right\}} \tag{5}$$

describing the dependence of energy (v) space structure on the eigenvalues κ_n , here we shall introduce analogous another function

$$\ln\mathscr{W}(v) = \int \frac{v^2}{v^2 v_{\mathrm{Te}}(3\sqrt{\pi}/2)G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3}\frac{\tau_{\mathrm{S}}}{\tau_{\mathrm{cx}}(v)}\mathrm{d}v \tag{6}$$

for the v-space structure affected the CX loss. By using them, the solution of the solubility condition Eq.(4) is given by

$$f_n(v) \propto \frac{1}{v^2 v_{\mathrm{Te}}(3\sqrt{\pi}/2)G(x_{\mathrm{e}}) + v_{\mathrm{c}}^3}\left\{\frac{\mathscr{V}(v)}{\mathscr{V}(v_{\mathrm{b}})}\right\}^{\kappa_n Z_2/3}\frac{\mathscr{W}(v)}{\mathscr{W}(v_{\mathrm{b}})}. \tag{7}$$

This modification on the v-space structure is valid also for the surface-averaged lowest Legendre order component $\left\langle \int_{-1}^{1} \overline{f}_{\mathrm{f}}\mathrm{d}\xi\right\rangle$ that is mentioned in Ref.3.

Next issue is how the formulas in Ref.4 for the surface-averaged 2nd Legendre order component $\left\langle \int_{-1}^{1} P_2(\xi)\overline{f}_{\mathrm{f}}\mathrm{d}\xi/B\right\rangle$ are modified. The adjoint equation method explained in that reference is useful even when the CX loss term is added to the usual Coulomb collision term since the relation $\int H\left\{(C_{\rm f}^{\rm A}-1/\tau_{\rm cx})F\right\}{\rm d}^3\mathbf{v}=\int F\left\{(C_{\rm f}-1/\tau_{\rm cx})H\right\}{\rm d}^3\mathbf{v}$ for arbitrary functions F,H is satisfied for the differential operators $C_{\rm f}$ in Eq.(1) and

$$C_{\rm f}^{\rm A}\equiv\frac{1}{\tau_{\rm S}}\left[-\frac{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{v^2}\frac{\partial}{\partial v}+\frac{Z_2v_{\rm c}^3}{v^3}\mathscr{L}\right] \tag{8}$$

even when the CX loss terms $-1/\tau_{\rm cx}(v)$ are added to them. Analogous to Ref.4, we should solve the ajoint equation $\left(V_{\|}+C_{\rm f}^{\rm A}-1/\tau_{\rm cx}\right)f_{\rm A}=H_2(v)P_2(\xi)/\tau_{\rm S}B(\theta,\zeta)$ when investigating the $\left\langle\int_{-1}^{1}P_2(\xi)\overline{f}_{\rm f}{\rm d}\xi/B\right\rangle$. The solution is obtained by using the relations

$$\begin{aligned}&(C_{\rm f}^{\rm A}-1/\tau_{\rm cx})P_2(\xi)\tfrac{\{\mathscr{V}(v)\}^{-Z_2}}{\mathscr{W}(v)}\int_0^v\tfrac{v^2H_2(v)\mathscr{W}(v)\{\mathscr{V}(v)\}^{Z_2}}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm d}v\\&=-\tfrac{1}{\tau_{\rm S}}P_2(\xi)H_2(v),\end{aligned} \tag{9}$$

$$\begin{aligned}&\tfrac{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{v^2}\tfrac{\{\mathscr{V}(v)\}^{-\alpha}}{\mathscr{W}(v)}\tfrac{\partial}{\partial v}\left[\mathscr{W}(v)\{\mathscr{V}(v)\}^{\alpha}F\right]\\&=\tfrac{\tau_{\rm S}}{\tau_{\rm cx}(v)}F+3\alpha\tfrac{v_{\rm c}^3}{v^3}F+\tfrac{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{v^2}\tfrac{\partial F}{\partial v},\end{aligned} \tag{10}$$

$$\begin{aligned}&\tfrac{\{\mathscr{V}(v)\}^{-\alpha}}{\mathscr{W}(v)}\int_0^v\tfrac{v^j\mathscr{W}(v)\{\mathscr{V}(v)\}^{\alpha}}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm d}v\\&\cong\tfrac{j+1}{j+1+3\alpha}\int_0^v\tfrac{v^j{\rm d}v}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm for}\,j\geq0,\alpha\geq0,{\rm and}\,v^3\ll v_{\rm c}^3,\end{aligned} \tag{11}$$

and

$$3v_{\rm c}^3\int\frac{\{\mathscr{V}(v)\}^{\alpha}{\rm d}v}{v\left\{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3\right\}}=\frac{\{\mathscr{V}(v)\}^{\alpha}}{\alpha}{\rm for}\,\alpha\neq0. \tag{12}$$

Because of Eq.(9), the adjoint equation an be rewritten as follows:

$$\begin{aligned}f_{\rm A}(\theta,\zeta,v,\lambda)&=-\tfrac{P_2(\xi)}{B}\tfrac{\{\mathscr{V}(v)\}^{-Z_2}}{\mathscr{W}(v)}\int_0^v\tfrac{v^2H_2(v)\mathscr{W}(v)\{\mathscr{V}(v)\}^{Z_2}}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm d}v+G_{\rm A}(\theta,\zeta,v,\lambda),\\\left(V_{\|}+C_{\rm f}^{\rm A}-1/\tau_{\rm cx}\right)G_{\rm A}&=v_{\|}\left(\mathbf{b}\cdot\nabla\tfrac{1}{B}\right)\tfrac{\{\mathscr{V}(v)\}^{-Z_2}}{\mathscr{W}(v)}\int_0^v\tfrac{v^2H_2(v)\mathscr{W}(v)\{\mathscr{V}(v)\}^{Z_2}}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm d}v.\end{aligned} \tag{13}$$

For the separated component $G_{\rm A}$, we shall use the usual asymptotic expansion method for the long mean free path condition $Z_2/(v_{\rm c}\tau_{\rm S})\ll\left|(\delta B/B)^{1/2}\mathbf{b}\cdot\nabla\ln B\right|$. The 0th order of $(v_{\rm c}\tau_{\rm S})^{-1}$ in the solution will have a form of

$$G_{\rm A}^0(\theta,\zeta,v,\lambda)=\left(\frac{1}{B}-\frac{1}{B_{\rm M}}\right)\frac{\{\mathscr{V}(v)\}^{-Z_2}}{\mathscr{W}(v)}\int_0^v\frac{v^2H_2(v)\mathscr{W}(v)\{\mathscr{V}(v)\}^{Z_2}}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}{\rm d}v+g_{\rm A}(v,\lambda). \tag{14}$$

The solubility condition $\left\langle\left(B/v_{\|}\right)\left(C_{\rm f}^{\rm A}-1/\tau_{\rm cx}\right)G_{\rm A}^0\right\rangle=0$ for the 1st order of $(v_{\rm c}\tau_{\rm S})^{-1}$ in the circulating pitch-angle $0\leq\lambda\leq1$ can be solved analytically when the CX loss term for as a minor component in the total solution is partly neglected and Eqs.(10-12) are used. The result for the 0th order of $(v_{\rm c}\tau_{\rm S})^{-1}$ is

$$\begin{aligned}&f_{\rm A}(\theta,\zeta,v_{\rm b},\lambda)=-\tfrac{1}{B_{\rm M}}\left(1-\tfrac{3}{2}\lambda\right)\int_0^{v_{\rm b}}\tfrac{v^2H_2(v)}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}\tfrac{\mathscr{W}(v)}{\mathscr{W}(v_{\rm b})}\left\{\tfrac{\mathscr{V}(v)}{\mathscr{V}(v_{\rm b})}\right\}^{Z_2}{\rm d}v\\&-\sum_n\tfrac{\left\langle(B_{\rm M}/B-1)\int_0^1\Lambda_n\left\{\partial(1-\lambda B/B_{\rm M})^{1/2}/\partial\lambda\right\}{\rm d}\lambda\right\rangle}{B_{\rm M}\left\langle\int_0^1\Lambda_n^2\left\{\partial(1-\lambda B/B_{\rm M})^{1/2}/\partial\lambda\right\}{\rm d}\lambda\right\rangle}\\&\times\Lambda_n(\lambda)\int_0^{v_{\rm b}}\tfrac{v^2H_2(v)}{v^2v_{\rm Te}(3\sqrt{\pi}/2)G(x_{\rm e})+v_{\rm c}^3}\\&\times\tfrac{\mathscr{W}(v)}{\mathscr{W}(v_{\rm b})}\left[\left\{\tfrac{\mathscr{V}(v)}{\mathscr{V}(v_{\rm b})}\right\}^{Z_2}+\tfrac{\kappa_n}{\kappa_n-3}\left\{\left\{\tfrac{\mathscr{V}(v)}{\mathscr{V}(v_{\rm b})}\right\}^{Z_2\kappa_n/3}-\left\{\tfrac{\mathscr{V}(v)}{\mathscr{V}(v_{\rm b})}\right\}^{Z_2}\right\}\right]{\rm d}v.\end{aligned} \tag{15}$$

By comparing these results with those in Refs.3-4, we can fined that, for the purpose of fast-ion-driven neoclassical phenomena, anisotropic pressure MHD equilibrium, and heating power calculations, the essential difference of the slowing down velocity distributions with and without the CX loss is the v-space reduction factor $\mathscr{W}(v)/\mathscr{W}(v_{\rm b})$. For using the FIDA as an experimental measurement of this reduction factor, the FIDASIM[1] calculation including this factor is now under preparation. An example of measured Balmer alpha spectrum in the Heliotron-J will be presented in the poster.

References

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