Energy confinement time in a magnetically confined thermonuclear fusion reactor

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Abstract

In the pursuit of viable nuclear fusion powered energy generation via magnetically confined fusion theory and experimentation, the single most important scientific question may be energy confinement time in a fusion plasma. In response, a simple theory is presented for quantitative calculations of the confinement times of plasma thermal energies in magnetically confined thermonuclear fusion reactors. The theory is based on radiation reaction associated with spontaneous electron cyclotron radiation as described by the Larmor formula. Good agreement is found between theory and experiment. An advanced Lawson criterion for ignition is derived, which is consistent with the latest magnetically confined fusion energy record achieved experimentally.

1. Introduction

There has been a stream of new fusion energy records achieved recently in thermonuclear fusion experiments[1,2]. In magnetically confined fusion, it is believed that the same modeling used in achieving new energy records predicts that the International Thermonuclear Experimental Reactor (ITER) will succeed[1]. The single most important scientific question regarding such success may be energy confinement time in a fusion plasma[3]. The energy confinement time of a plasma is defined as the thermal energy content of the plasma divided by the power loss, i.e., $\tau_E \equiv W/P_{loss}$, where W is the thermal energy of the plasma, and P_{loss} is the power loss. The energy confinement time as defined is not necessarily sustaining time, which can be indefinite with sufficient supplied heating power. Until this article, there has not been a simple theory for τ_E calculation. The fusion research community relies on derived empirical scaling laws for energy confinement times in fusion reactor designs[4].

2. Theoretical Model

In a magnetically confined plasma, an electron gyrates in the magnetic field and spontaneously emits cyclotron radiation. It loses its perpendicular kinetic energy $E_{e\perp}$ via cyclotron radiation emission according to the Larmor formula. In the leading order of approximation, the electrons in the plasma are isothermal, and the total kinetic energy of an electron on average is $E_e \approx 3E_{e\perp}/2$. The ions and electrons tend to thermalize among themselves on a time scale shorter than or comparable to the characteristic time scale over which the electrons lose their energies via cyclotron radiation, such that $E_i \approx E_e = E$, where E_i is the total kinetic energy of an ion on average. The energy loss rate of an ion is about the same as that of an electron. It is readily shown that the confinement time of plasma thermal energy is simply given by[5]

$$\tau_E \equiv -\frac{E}{dE/dt} = \frac{3c}{4\omega_c^2 r_e} = \frac{2.6\ \mathrm{s}}{B_{\mathrm{Tesla}}^2} \tag{1}$$

where c is the speed of light, $r_e = 2.8 \times 10^{-15}$ m is the classical electron radius, $\omega_c = eB/m_e$ is the electron cyclotron frequency, and B is the magnetic field.

The importance of spontaneous electron cyclotron radiation in energy confinement has not received careful attention in fusion research for several decades. Spontaneous electron cyclotron radiation belongs to the category of bremsstrahlung radiation. It is quite discrete in the radiation spectrum. In burning plasmas, wherein most of the plasma heating comes from fusion reactions, the transport and net loss of cyclotron radiation has not been fully understood[6]. In the parameter regime of interest to fusion, the discrete-spectrum spontaneous cyclotron radiation may be stronger than the broad-spectrum bremsstrahlung radiation due to the Rutherford scattering, although the cyclotron radiation friction is small compared with the dynamical friction[7]. As an example of spontaneous electron cyclotron radiation for parameters close to the ITER design, at magnetic field $B = 5$ T, plasma density $n_e = n_i = 1 \times 10^{20}$ m^{-3}, and plasma temperature $k_B\mathrm{T} = 10$ keV, the cyclotron radiation power per electron is 1.5×10^{-14} W, whereas the bremsttrahlung radiation power per electron is 1.6×10^{-16} W. More importantly, the photons emitted via spontaneous electron cyclotron radiation are incoherent and they are not in thermal equilibrium with the plasma. Thus, they are unlikely to be reabsorbed by the electrons in the plasma because the Compton cross section and wavelength are too small. Ultimately, they are lost in the reactor chamber wall.

3. Comparison between theory and experiment

For comparison between theory and experiment, it is important to note that confinement time is different from sustaining time, which can be indefinite with sufficient supplied heating power. It is essential to use high quality data showing how plasma parameters vary after supplied heating power is turned off. Figure 1 shows comparison between theory and experiment.

At the Tokamak Fusion Test Reactor (TFTR) in a normal-conducting tokamak configuration, heating was achieved by injecting neutral beams into a deuterium-tritium (D-T) plasma, creating nearly optimal conditions for D-T nuclear fusion[3]. The experimental

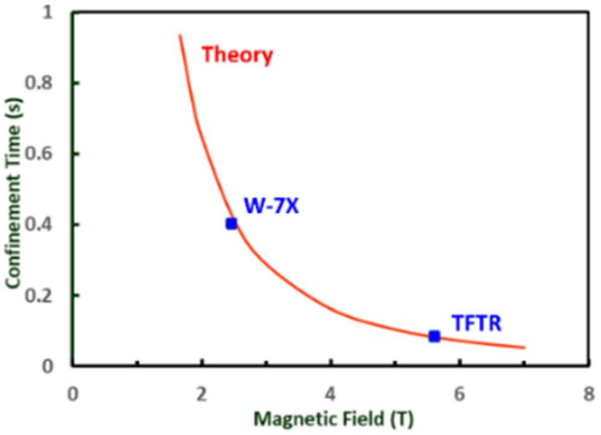

Fig. 1 Theory versus experiment chart.

measurements shown in Fig. 7 of Ref. 3 allow an indirect, *qualitative* comparison between theory and experiment. The fusion power decreased rapidly after the neutral beam heating power was turned off at t = 2.85 s. The time scale over which the fusion power decreased is estimated to be $\tau_E = 0.08$ s. which is in qualitative agreement with the theoretical τ_E = 0.083 s for a toroidal magnetic field of 5.6 T in the tokamak.

At Wendelstein-7X (W-7X) in a superconducting stellarator configuration, electron cyclotron resonance heating (ECRH) was employed as a primary source to heat a hydrogen plasma[8]. The experimental measurements shown in Fig. 8 of Ref. 8 allow a *quantitative* comparison between theory and experiment. After the ECRH was turned off at t = 4.5 s, both the measured electron temperature and the measured ion temperature decreased. The electron temperature decreased slightly faster than the ion temperature. The average of the time scales over which the electron and ion temperatures dropped off is estimated to be τ_E = 0.04 s, which is in quantitative agreement with the theoretical τ_E = 0.042 s for the magnetic field of 2.5 T at the center of the stellarator.

4. Advanced Lawson's criterion for ignition

Substituting the energy confinement time τ_E in Eq. (1) in the Lawson criterion for ignition[9], we arrive at an advanced Lawson criterion. At the optimal temperature of 14 keV for D-T fusion, the advanced Lawson criterion for ignition becomes simply[5]

$$\beta \equiv \frac{p}{p_{mag}} \geq 0.92 = 92\% \tag{2}$$

with the so-called β parameter measuring the thermal pressure $p = 2nk_BT$ relative to the magnetic pressure $p_{mag} = B^2/2\mu_0$. D-T ignition corresponds to $Q \geq 5$, where Q is fusion energy relative to supplied heating energy. It follows from Eq. (2) that the upper limit of fusion energy gain is $Q_{Limit} \approx 5\beta/0.92 = 5.4\beta$.

At present, the achievable β is limited to a few percent in stellarator and tokamak experiments. In stellarator experiments, the volume-averaged β up to 5.1% has been achieved[10]. For a typical tokamak with a ratio of major radius to minor radius of $R/a = 3$, the magnetohydrodynamics (MHD) stability limit is $\beta = a/R[6.25 + (a/R)^2] = 5.2\%$[4]. The latest D-T fusion energy record[1] achieved experimentally at the Joint European Torus (JET) with sustained $Q = 0.33$ for 5 s appears to be consistent with the advanced Lawson criterion. Indeed, the theoretical limit is $Q_{Limit} = 0.35$ taking the JET geometric parameters $R = 3$ m and $a = 1.25$ m and the associated MHD stability limit $\beta = 6.4\%$.

5. Conclusion

A simple theory was presented for energy confinement in a magnetically confined thermonuclear fusion reactor. Good agreement was found between theory and experiment. An advanced Lawson criterion for ignition was derived. The latest magnetically confined fusion energy record achieved experimentally was found to be consistent with this advanced Lawson criterion. If this advanced Lawson criterion continues to predict the performance of magnetically confined fusion experiments as more data points are added in the comparison between theory and experiment, it will likely have profound implications for fusion research. Because experimentally achievable plasma pressure relative to magnetic pressure is only a few percent in a tokomak or a stellarator, it is predicted to be very difficult to generate net positive fusion energy in such reactors considering the confinement time challenges presented by spontaneous electron cyclotron radiation, as indicated by this newly derived advanced Lawson criterion.

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